CAAM 436
Partial Differential Equations I
Syllabus, Fall 1997

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Purpose: To derive many of the partial differential equations of mathematical physics from first principles, and to introduce modern methods for constructing solutions of these equations.

Plan: The course consists of two parts. In the first part we combine basic physical principles with vector calculus to derive many important PDEs governing motion of fluids and solids. We treat the regularity of solutions and other functions cavalierly; the point is to understand where all of these famous equations come from. In the second part, we go about solving the Poisson problem, which appears as a special case of every one of the equations derived in the first part. The approach is "modern", i.e. applies to a wide range of problems and leads both to a theoretical construction of the solution and to an important and widely used numerical technique, the finite element method. Perhaps surprisingly, regularity of the solution emerges as a central issue, and we are led naturally to develop basic ideas from functional analysis.

Continuation: The second semester course, CAAM 437: Partial Differential Equations II, will cover additional topics in the same spirit, including time dependent problems such as heat flow and wave motion.

Prerequisites: Advanced calculus / introductory real analysis, CAAM 321/322, or Math 321 and some sort of course on rigorous vector calculus (which is what CAAM 322 is, and occasionally what Math 322 is). Linear algebra at the level of CAAM 310 or Math 355 is helpful.

Text: This course has a web page:

`http://www.caam.rice.edu/~caam436/index.html`

on which I will post course notes, homework assignments, and solution sets. There is no (other) required text.

Recommended Reading: While the course has no required text, an excellent reference for the first part is Morton E. Gurtin's *An Introduction to Continuum Mechanics*. This book will be held on 1 day reserve in Fondren Library, and I am willing to loan out my personal copy as well. It may be worth purchase for those with sufficient interest, though it is expensive. I will suggest other reading appropriate to various topics as they arise.

Grading: I will assign frequent homeworks, as the material on which they are based is covered in class. Your grade will be based on your homework performance.

Course Outline:

1. Partial Differential Equations of Continuum Mechanics
    (a) Conservation principles and Transport
    (b) Strain
    (c) Stress
    (d) Ideal and elastic fluids, Euler's equation
    (e) Small fluid motions and acoustic waves
    (f) Newtonian fluids, Navier-Stokes equations
    (g) Elastic solid motion
    (h) Small elastic motions and linear elastodynamics
2. The Poisson problem - a special case of everything
    (a) sets of functions as vector spaces, the spaces C^k and C_0^k, the Laplace operator as a linear map
    (b) the role of boundary conditions and physical examples
    (c) the L^2 inner product and symmetric boundary value problems
    (d) linear equations and quadratic forms - variational form of symmetric BVP
    (e) approximation by finite dimensional subspaces - examples, the crisis of convergence
    (f) completeness, Hilbert space, completion of inner product spaces
    (g) examples of Hilbert space: L^2, the Sobolev spaces H^k
    (h) the Lax-Milgram theorem: weak solution of the Poisson problem
    (i) approximation in H^1 by piecewise linear functions, the Finite Element Method
    (j) weak solutions are solutions