Array Imaging in Random Media
Part I

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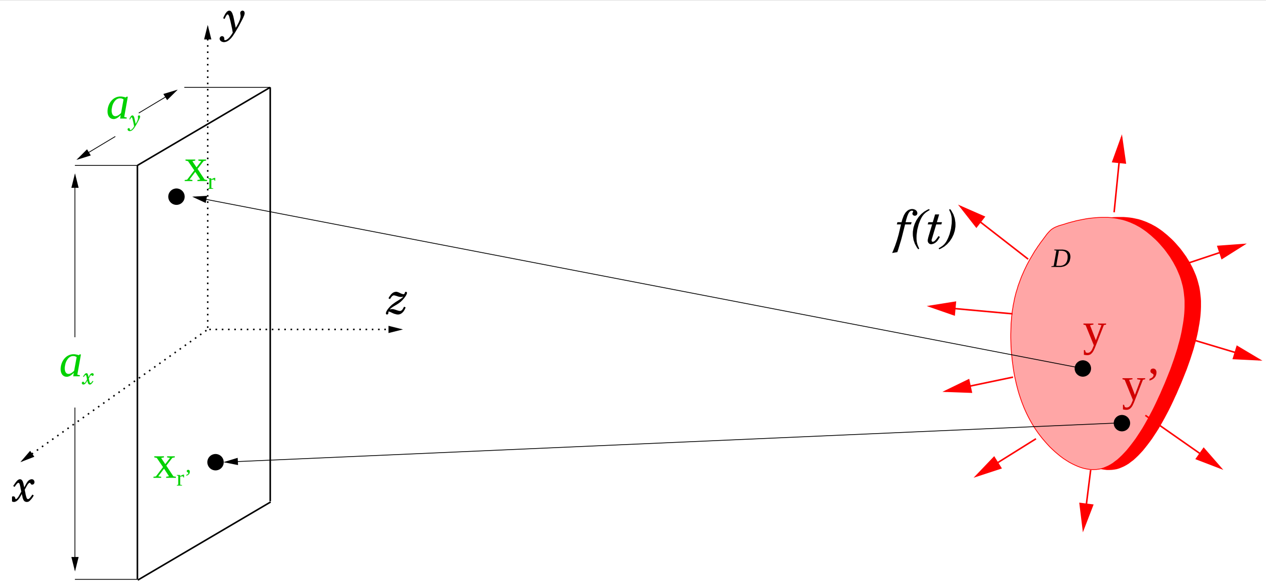
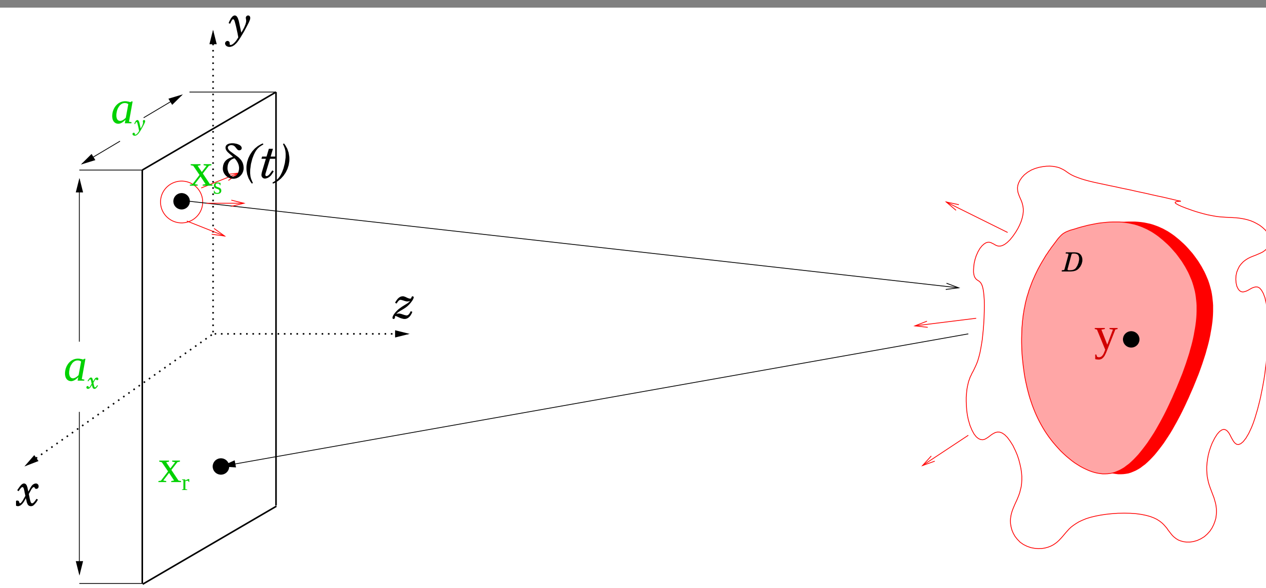
Passive Array Imaging in Clutter



- Array data: $P(\mathbf{x}_r, t)$ for $(\mathbf{x}_r, t)$ a set of receiver locations in $\mathbb{R}^2$ and time in $\mathbb{R}_+$.
- Object: continuous distribution of sources in $\mathcal{D}$
- Goal: recover $\mathcal{D}$ in cluttered background.

Active Array Imaging in Clutter



- Array data: $\Pi(\mathbf{x}_s, \mathbf{x}_r, t)$ for $(\mathbf{x}_s, \mathbf{x}_r, t)$ a set of source and receiver locations in $\mathbb{R}^2$ and time in $\mathbb{R}_+$.
- Object: an extended scatterer with support in $\mathcal{D}$
- Goal: recover $\mathcal{D}$ in cluttered background.
- Applications: Non-destructive testing, seismic, sonar and broadband radar imaging.

What is the clutter?

We assume that the background velocity $c(\mathbf{x})$ consists of

- a smooth part $c_o(\mathbf{x})$, that is known or can be estimated
- and the clutter: inhomogeneities that cannot be precisely estimated

We model the clutter as a random process $\longrightarrow$

What is the clutter?

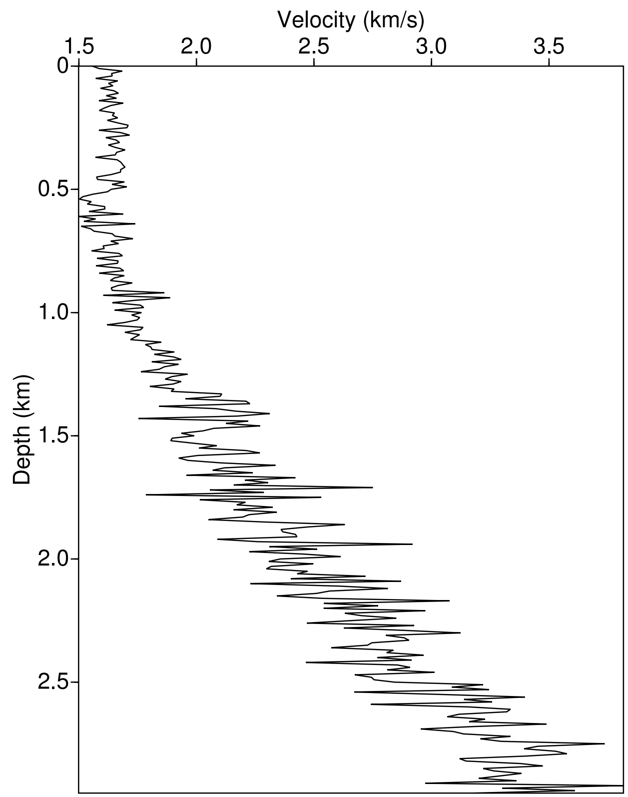
We write the index of refraction $n(\mathbf{x}) = c_0/c(\mathbf{x})$ as

$$n^2(\mathbf{x}) = n_0^2(\mathbf{x})\left(1 + \sigma\mu\left(\frac{\mathbf{x}}{\ell}\right)\right)$$

- $n_0(\mathbf{x})$: smooth and known ($n_0(\mathbf{x}) = 1$ in the numerics).
- μ: statistically homogeneous random process with mean zero and rapidly decaying correlation
- ℓ: correlation length (scale of the inhomogeneities)
- σ: strength of the fluctuations

Example of clutter

- simulated velocity profile in a well log



courtesy of Eric Dussaud

Modeling the clutter

The random process μ is real valued with

- mean zero, $< \mu >= 0$
- correlation function:

$$R(\mathbf{x}_1, \mathbf{x}_2) =< \mu(\mathbf{x}_1)\mu(\mathbf{x}_2) >$$

or by introducing $\overline{\mathbf{x}} = \dfrac{\mathbf{x}_1 + \mathbf{x}_2}{2}, \widetilde{\mathbf{x}} = \mathbf{x}_2 - \mathbf{x}_1$

$$R(\overline{\mathbf{x}}, \widetilde{\mathbf{x}}) =< \mu(\overline{\mathbf{x}} - \widetilde{\mathbf{x}}/2)\mu(\overline{\mathbf{x}} + \widetilde{\mathbf{x}}/2) >$$

- and we assume that the correlation function depends only on the distance

$$R(\overline{\mathbf{x}}, \widetilde{\mathbf{x}}) = R(\widetilde{\mathbf{x}})$$

Synthetic realization of random media

- on a rectangular grid we generate a filter $F(\mathbf{x})$
- we compute the Fourier transform $\hat{F}(\mathbf{k})$ of $F(\mathbf{x})$
- we generate a white noise distribution $\hat{W}(\mathbf{k})$ ($< \hat{W} >= 0$, std=1, $< \overline{\hat{W}(\mathbf{k}_1)}\hat{W}(\mathbf{k}_2) >= \delta(\mathbf{k}_1 - \mathbf{k}_2)$)
- we compute $\mu(\mathbf{x}) = \mathcal{F}^{-1}(\hat{W}\hat{F})$
- the correlation function of $\mu(\mathbf{x})$ is

$$R(\widetilde{\mathbf{x}}) = (2\pi)^{-d} \int d\mathbf{k} e^{i\mathbf{k}\cdot\widetilde{\mathbf{x}}} \overline{\hat{F}(\mathbf{k})} \hat{F}(\mathbf{k})$$

- we chose F so as to obtain the desired R

Synthetic realization of random media

Examples of isotropic clutter correlation functions

- Gaussian

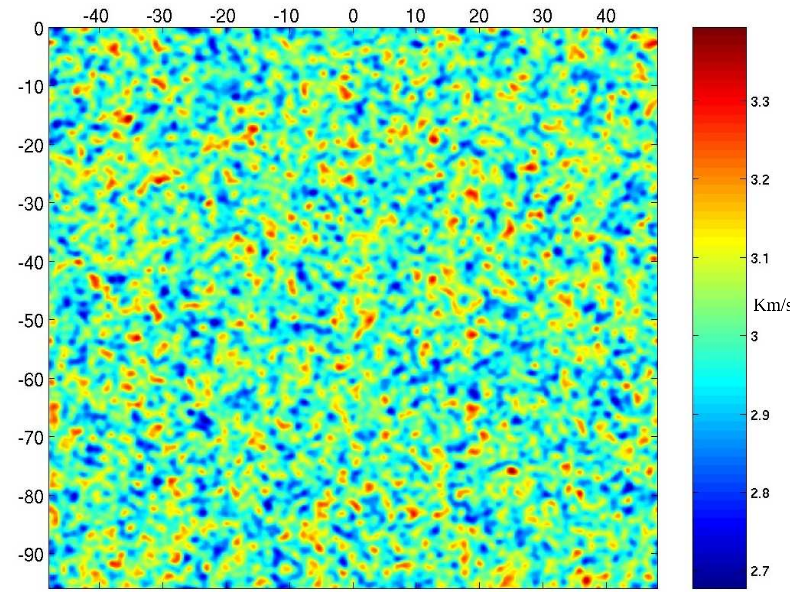
$$R(|\mathbf{x}_1 - \mathbf{x}_2|) = e^{-\frac{|\mathbf{x}_1 - \mathbf{x}_2|^2}{2\ell^2}}$$

- Power low

$$R(|\mathbf{x}_1 - \mathbf{x}_2|) = (1 + \frac{|\mathbf{x}_1 - \mathbf{x}_2|}{\ell})e^{-\frac{|\mathbf{x}_1 - \mathbf{x}_2|}{\ell}}$$

Synthetic realization of random media

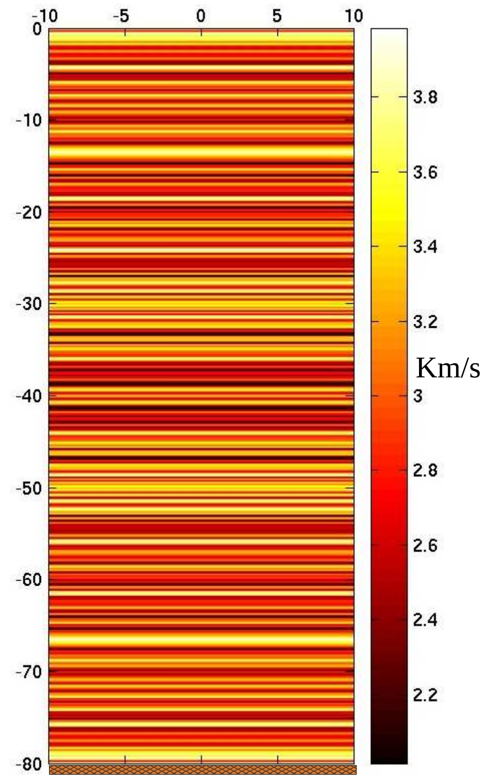
- 2D example with gaussian correlation fct



- here the correlation length ℓ is the same in all directions of propagation.

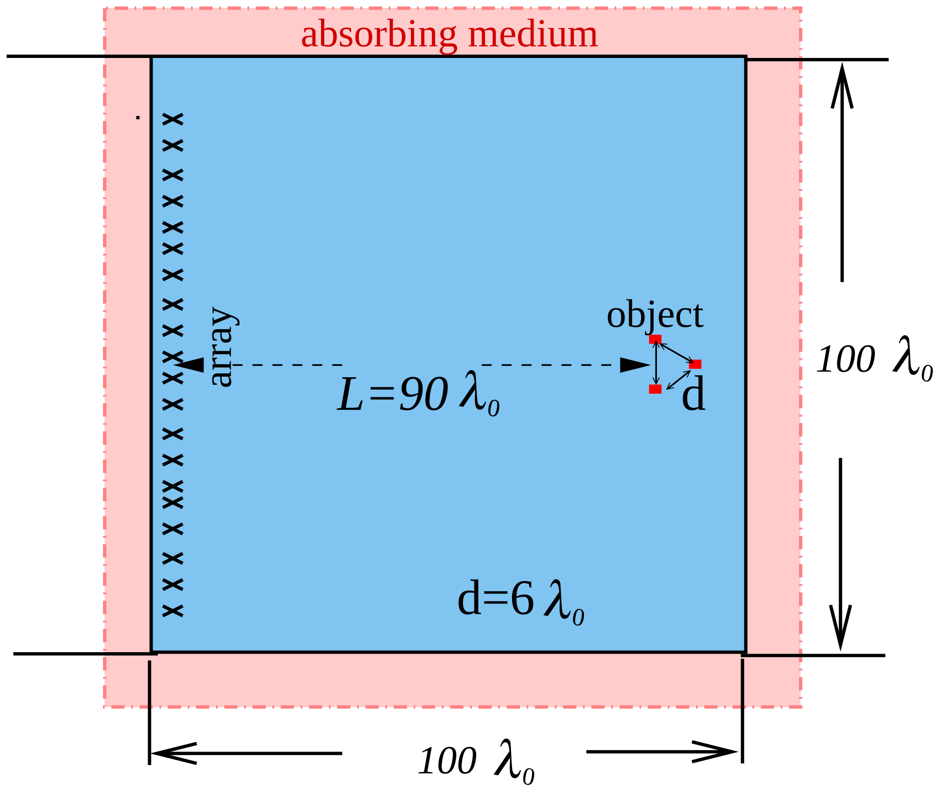
Synthetic realization of random media

- 1D example : Anisotropic (layered) clutter



- here the correlation length is infinite in one direction and finite in the other one.

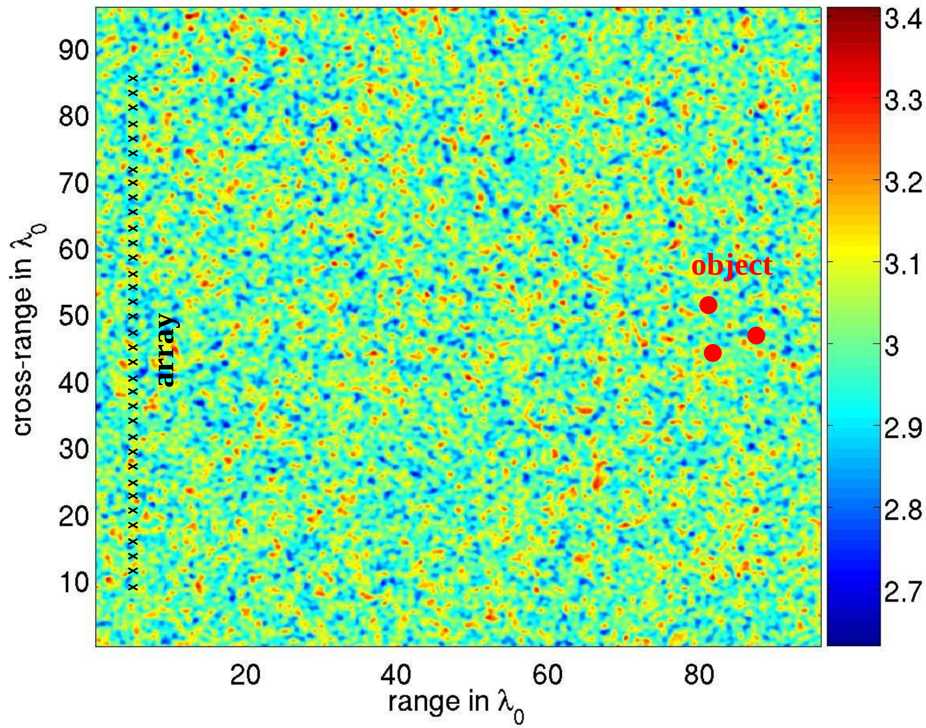
Setup for the numerical simulations



In the context of ultrasound non-destructive testing

- Frequency range $150 - 450$KHz, $c_0 = 3$Km/s, $\lambda_0 = 1$cm.
- Linear array with $N = 181$ elements of aperture $a = 90\lambda_0$. The range is $L = 90\lambda_0$. The objects are disks with diameter λ_0 (Dirichlet).

Setup for the numerical simulations



For the clutter:

- μ has a Gaussian correlation function
$$R(\mathbf{x},\mathbf{x}') = < \mu(\mathbf{x})\mu(\mathbf{x}') > = R(|\mathbf{x}-\mathbf{x}'|) = e^{-\frac{|\mathbf{x}-\mathbf{x}'|^2}{2\ell^2}}$$
- with $\ell = 0.5\lambda_0$ and $\sigma = 0.03$

Setup for the numerical simulations

Data obtained by solving the wave equation in time and 2D space (FEM + PML)

$$\varrho(\mathbf{x})\frac{\partial \mathbf{v}}{\partial t} + \nabla p = 0$$

$$\kappa(\mathbf{x})\frac{\partial p}{\partial t} + \mathrm{div}\mathbf{v} = f(t)\delta(\mathbf{x} - \mathbf{x}_s)$$

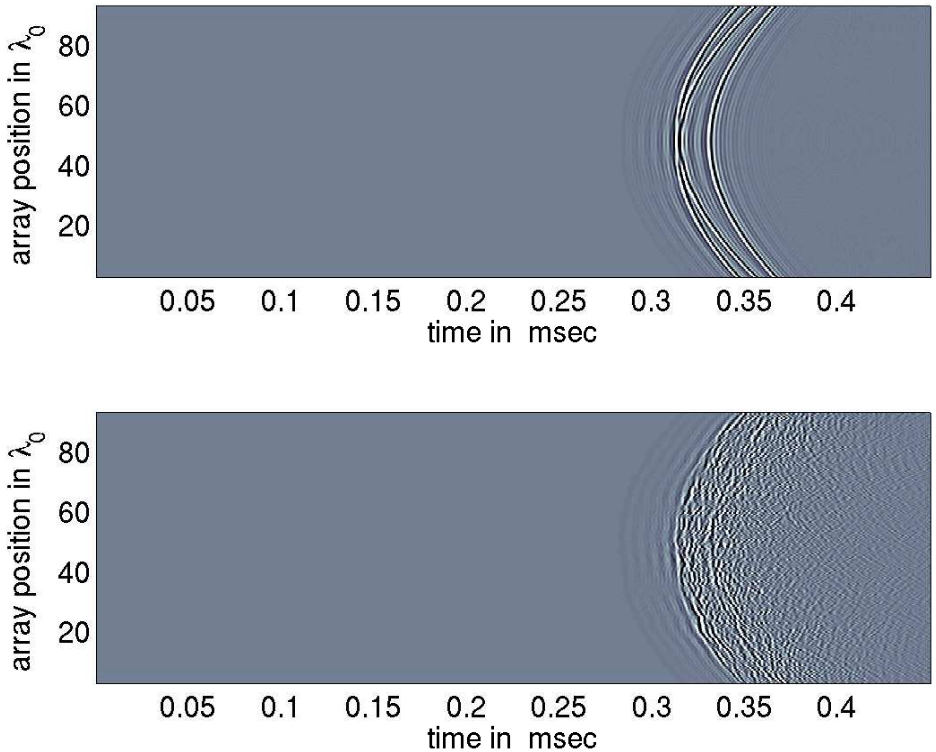
$$\varrho(\mathbf{x}) = 1$$

$$\kappa(\mathbf{x}) = \frac{1}{\varrho c^2(\mathbf{x})}$$

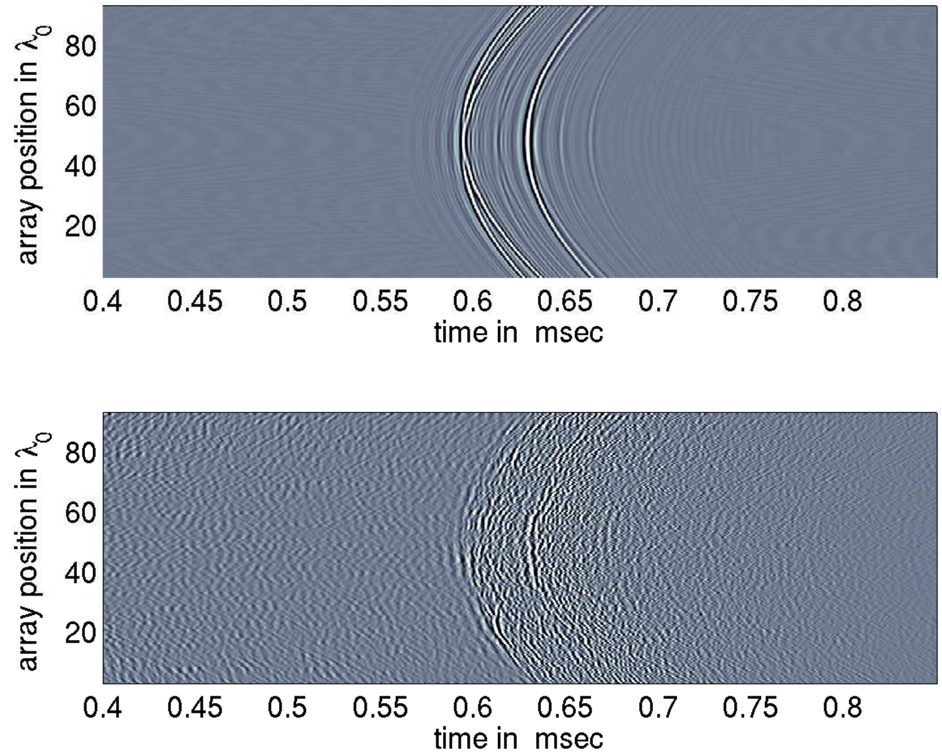
- We need to resolve all the scales involved (wavelength $\sim$ correlation length) $\rightarrow$ heavy computations
- For the active array case we have 3000×3000 points resulting to $9M \times 3$ unknowns and about 13000 iterations in time.

Data on the array: traces

Passive array



Active array



The clutter impedes the imaging process as the significant multipathing of the waves by the inhomogeneities results to noisy data traces (the noise is not simply additive)

Migration imaging

Passive array: imaging functional for search point $\mathbf{y}^S$

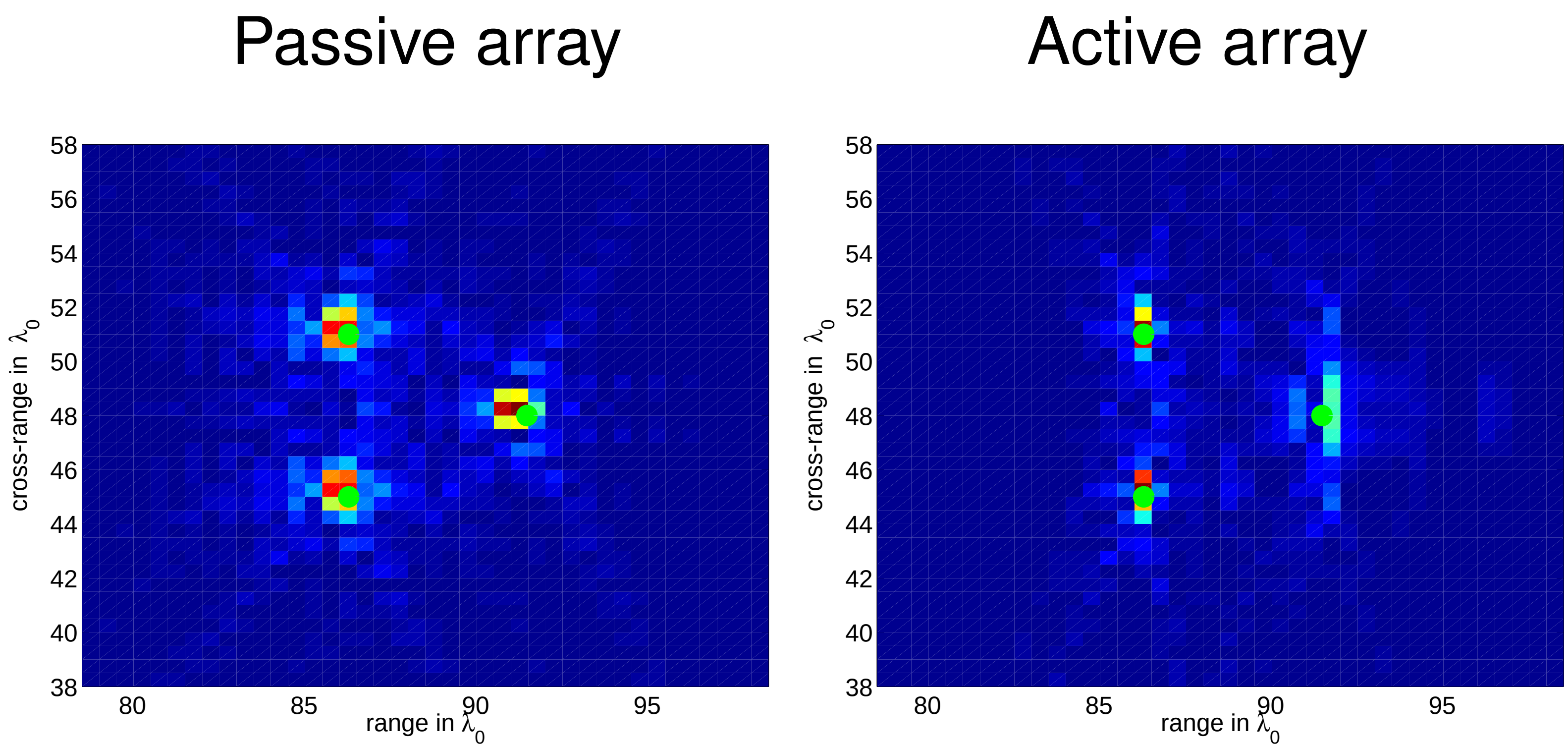
$$\mathcal{I}^{\text{KM}}(\mathbf{y}^s) = \sum_r P(\mathbf{x}_r, \tau(\mathbf{x}_r, \mathbf{y}^S)) = \sum_r \int_B \frac{d\omega}{2\pi} \hat{P}(\mathbf{x}_r, \omega) \overline{G_0(\mathbf{x}_r, \mathbf{y}^s, \omega)}$$

- with $G_0(\mathbf{x}_s, \mathbf{y}^s, \omega) = e^{i\omega\tau(\mathbf{x}_s, \mathbf{y}^s)}$ and $\tau(\mathbf{x}, \mathbf{y}) = |\mathbf{x} - \mathbf{y}|/c_0$ the travel time in the known smooth background (here homogeneous)

Active array: imaging functional for one source

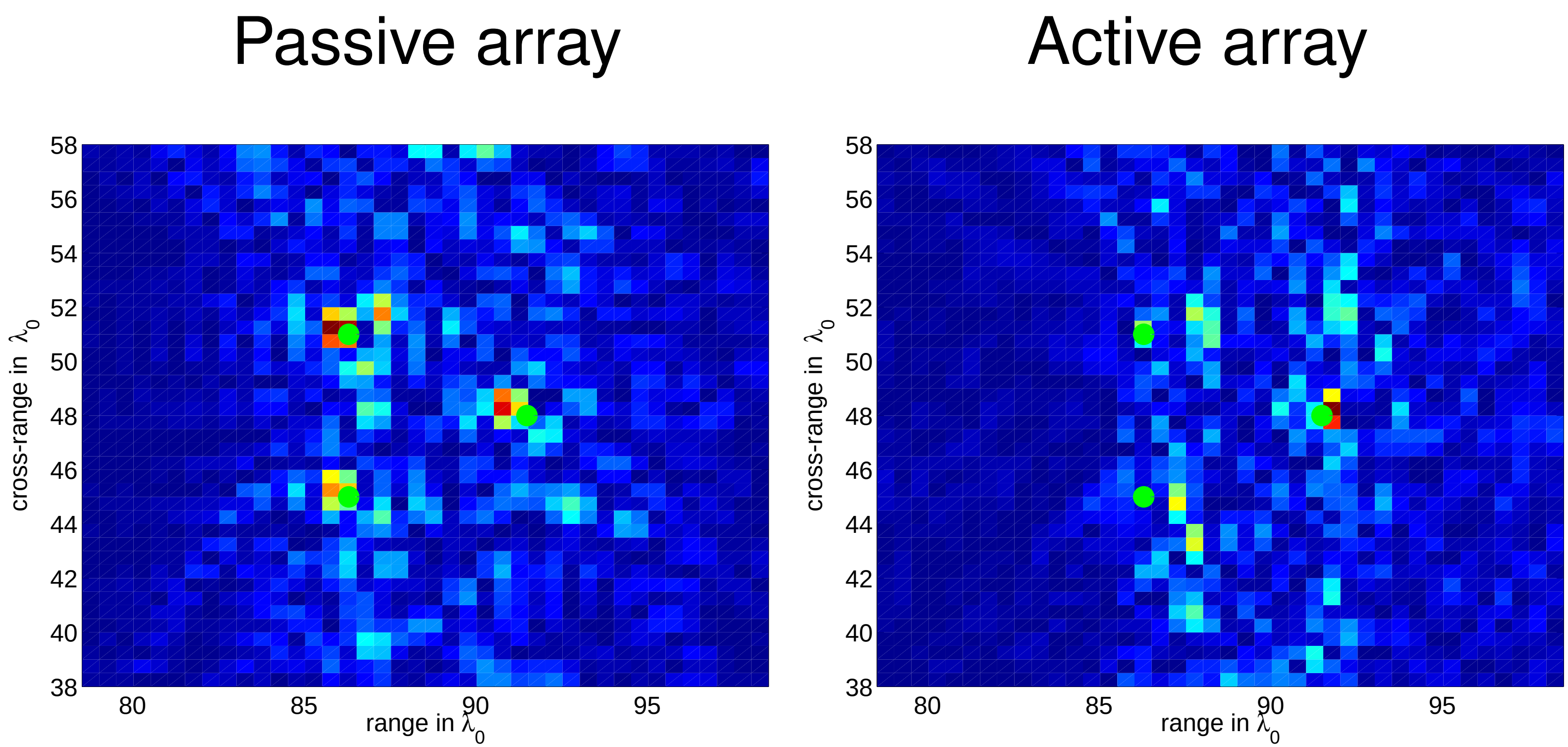
$$\begin{aligned} \mathcal{I}^{\text{KM}}(\mathbf{y}^s) &= \sum_{r=1}^{N_r} P(\mathbf{x}_s, \mathbf{x}_r, \tau(\mathbf{x}_s, \mathbf{y}^s) + \tau(\mathbf{x}_r, \mathbf{y}^s)) \\ &= \sum_{r=1}^{N_r} \int \frac{d\omega}{2\pi} \hat{P}(\mathbf{x}_s, \mathbf{x}_r, \omega) \overline{G_0(\mathbf{x}_s, \mathbf{y}^s, \omega) G_0(\mathbf{x}_r, \mathbf{y}^s, \omega)} \end{aligned}$$

Migration results



- length is scaled by λ_0
- the search domain is a square $20\lambda_0 \times 20\lambda_0$ centered at the objects
- the pixel size is $\lambda_0/2$.

Migration results



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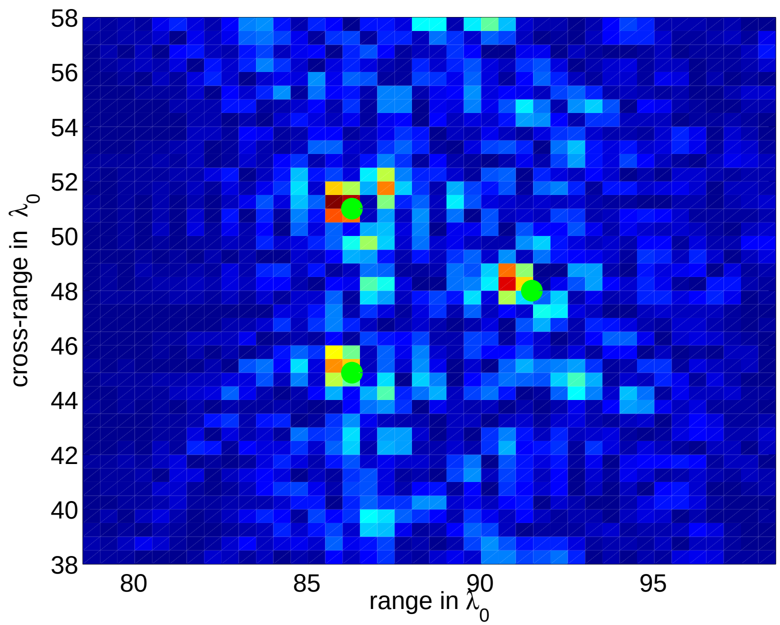
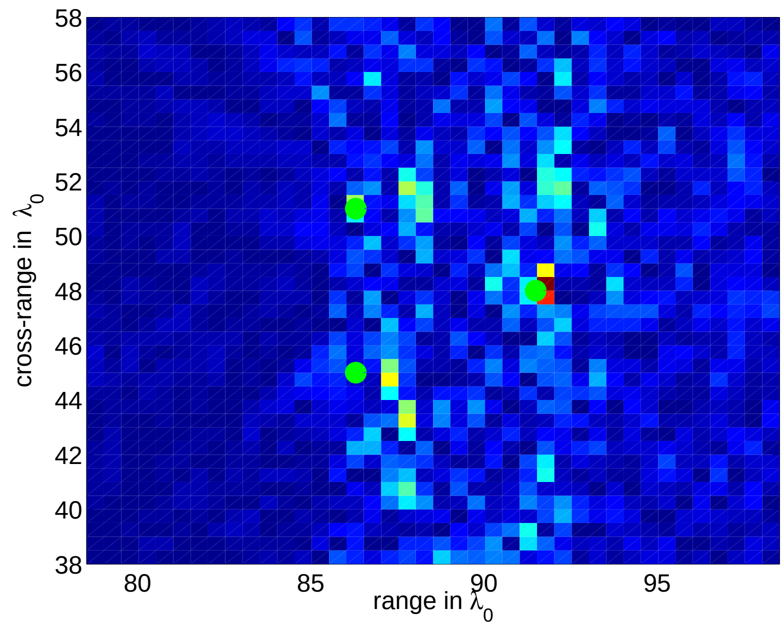
Migration results

Passive array



Active array



- Classical migration does not work in clutter. To make migration work we should remove the delay spread:
  - ✗ trace denoising ? (noise is not additive)
  - ✔ we use the coherent interferometry (CINT)

Coherent interferometry (CINT)

- we cross-correlate the traces locally in space and time:
  - cross-correlation in space is limited by the decoherence length X_d
  - cross-correlation in time is limited by the delay spread T_d
- we call these local cross-correlations coherent interferograms
- CINT consists in migrating the coherent interferograms to the search point $\mathbf{y}^s$ using $G_0(\mathbf{x}_r, \mathbf{y}^s, \omega)$

CINT imaging functional

For one source located at $\mathbf{x}_s$, we compute

$$\mathcal{I}^{\text{CINT}}(\mathbf{y}^s; \Omega_d, \kappa_d) \sim$$

$$\int d\omega \int_{|\omega-\omega'|\leq\Omega_d} d\omega' \sum \sum_{r,r' \in \mathcal{X}\left(\frac{\omega+\omega'}{2}, \kappa_d\right)} \hat{Q}(\mathbf{x}_r, \mathbf{x}_s, \omega; \mathbf{y}^s)\overline{\hat{Q}(\mathbf{x}_{r'}, \mathbf{x}_s, \omega'; \mathbf{y}^s)}$$

with $\hat{Q}(\mathbf{x}_r, \mathbf{x}_s, \omega; \mathbf{y}^s) = \hat{\Pi}(\mathbf{x}_r, \mathbf{x}_s, \omega) e^{-i\omega[\tau(\mathbf{x}_s, \mathbf{y}^s) + \tau(\mathbf{x}_r, \mathbf{y}^s)]}$.

CINT imaging functional

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$$\mathcal{I}^{\text{CINT}}(\mathbf{y}^s; \Omega_d, \kappa_d) \sim$$

$$\int d\omega \int_{|\omega-\omega'|\le\Omega_d} d\omega' \sum_{r,r'\in\mathcal{X}\left(\frac{\omega+\omega'}{2},\kappa_d\right)} \hat{Q}(\mathbf{x}_r, \mathbf{x}_s, \omega; \mathbf{y}^s)\overline{\hat{Q}(\mathbf{x}_{r'}, \mathbf{x}_s, \omega'; \mathbf{y}^s)}$$

- we cross-correlate nearby frequencies $|\omega - \omega'| \le \Omega_d$, with Ω_d the decoherence frequency $(\sim 1/T_d)$
- and nearby receivers $\mathcal{X}(\omega, \kappa_d) = \left\{r, r' = 1, \ldots, N; \;\; |\mathbf{x}_r - \mathbf{x}_{r'}| \le X_d(\omega) = \frac{c_o}{\omega \kappa_d}\right\}.$
- Ω_d and κ_d are clutter-dependent coherence parameters that must be estimated from the data.

CINT imaging functional

For one source located at $\mathbf{x}_s$, we compute

$$\mathcal{I}^{\text{CINT}}(\mathbf{y}^s; \Omega_d, \kappa_d) \sim$$

$$\int d\omega \int_{|\omega-\omega'|\leq\Omega_d} d\omega' \sum \sum_{r,r'\in\mathcal{X}\left(\frac{\omega+\omega'}{2},\kappa_d\right)} \hat{Q}(\mathbf{x}_r, \mathbf{x}_s, \omega; \mathbf{y}^s)\overline{\hat{Q}(\mathbf{x}_{r'}, \mathbf{x}_s, \omega'; \mathbf{y}^s)}$$

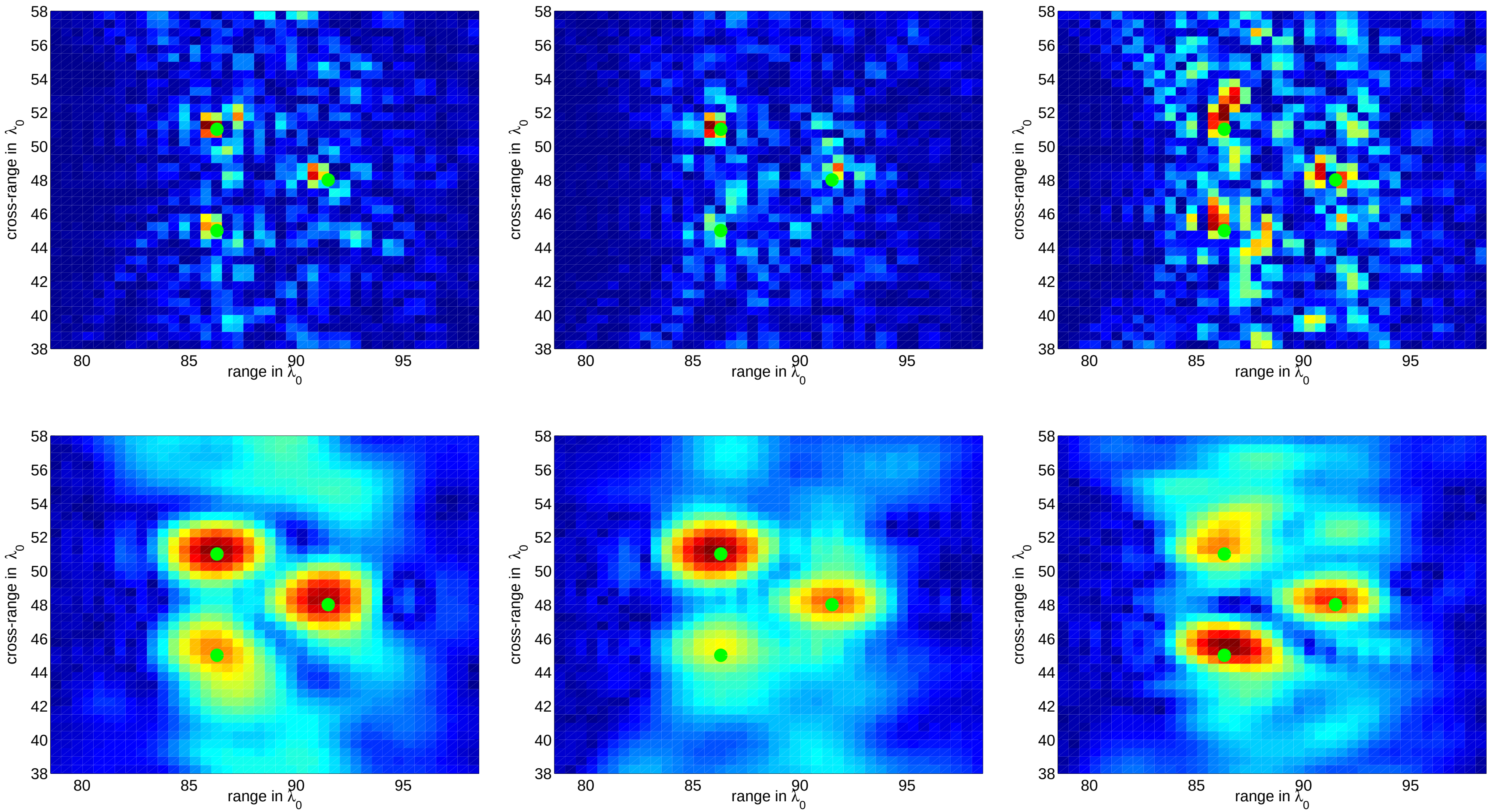
- CINT can be also viewed as a statistically smoothed migration.
- The smoothing of the image depends on the decoherence parameters.

Adaptive Selection of κ_d and Ω_d

- How can we find Ω_d and κ_d ?
- We can derive (theoretical) formulae for Ω_d and κ_d. This is model dependent. (L. Borcea, G. Papanicolaou, CT, *Interferometric array imaging in clutter*, Inverse Problems, 2005.)
- The decoherence parameters can be estimated adaptively during the image formation process. (L. Borcea, G. Papanicolaou, CT, *Adaptive interferometric imaging in clutter and optimal illumination*, Inverse Problems, 2006.)
- Lets assume for the moment that we know the "optimal" Ω_d^* and κ_d^*.

CINT results

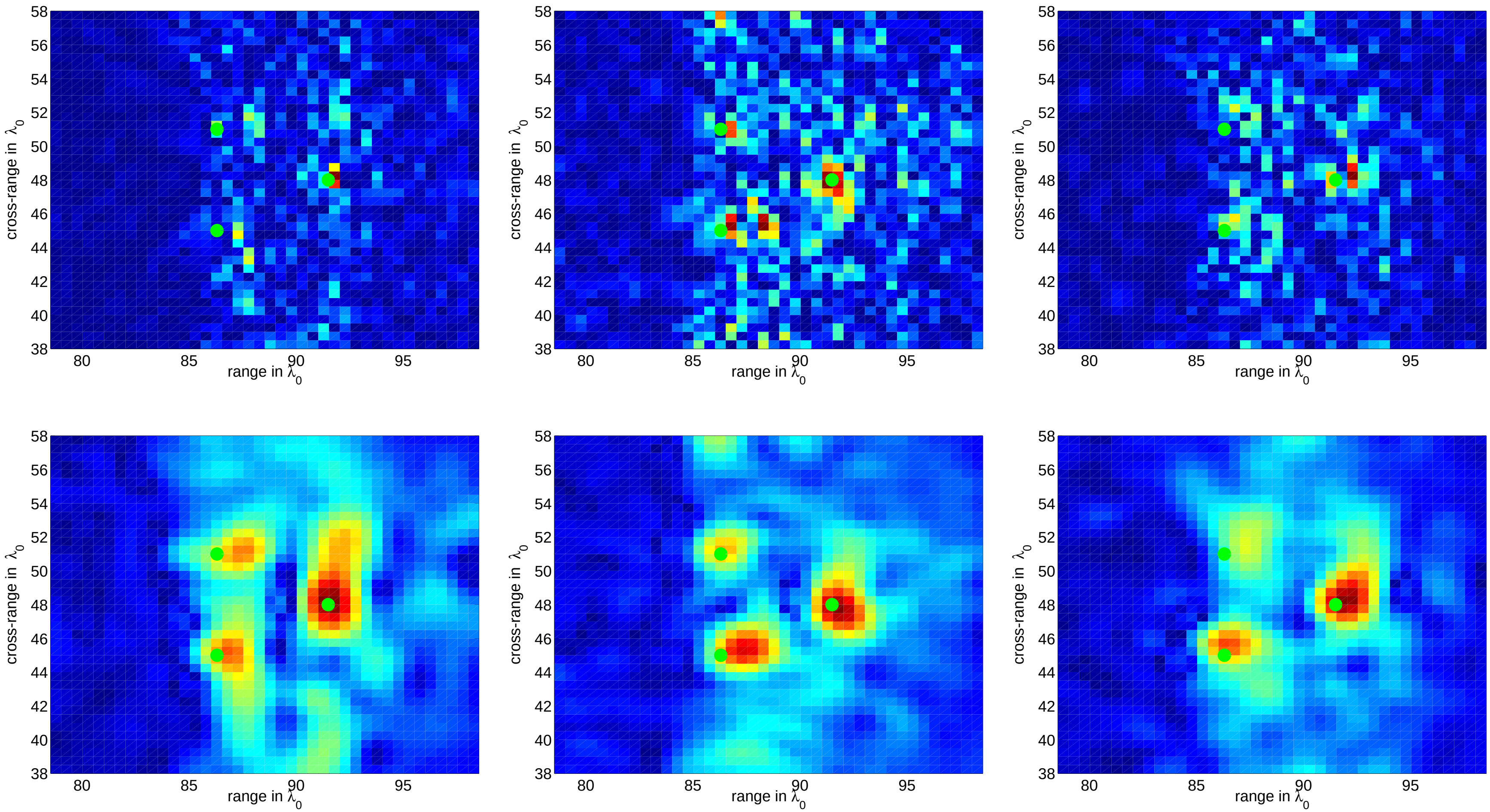
Passive array



Results for three different realizations of the clutter. Top: migration. Bottom: CINT

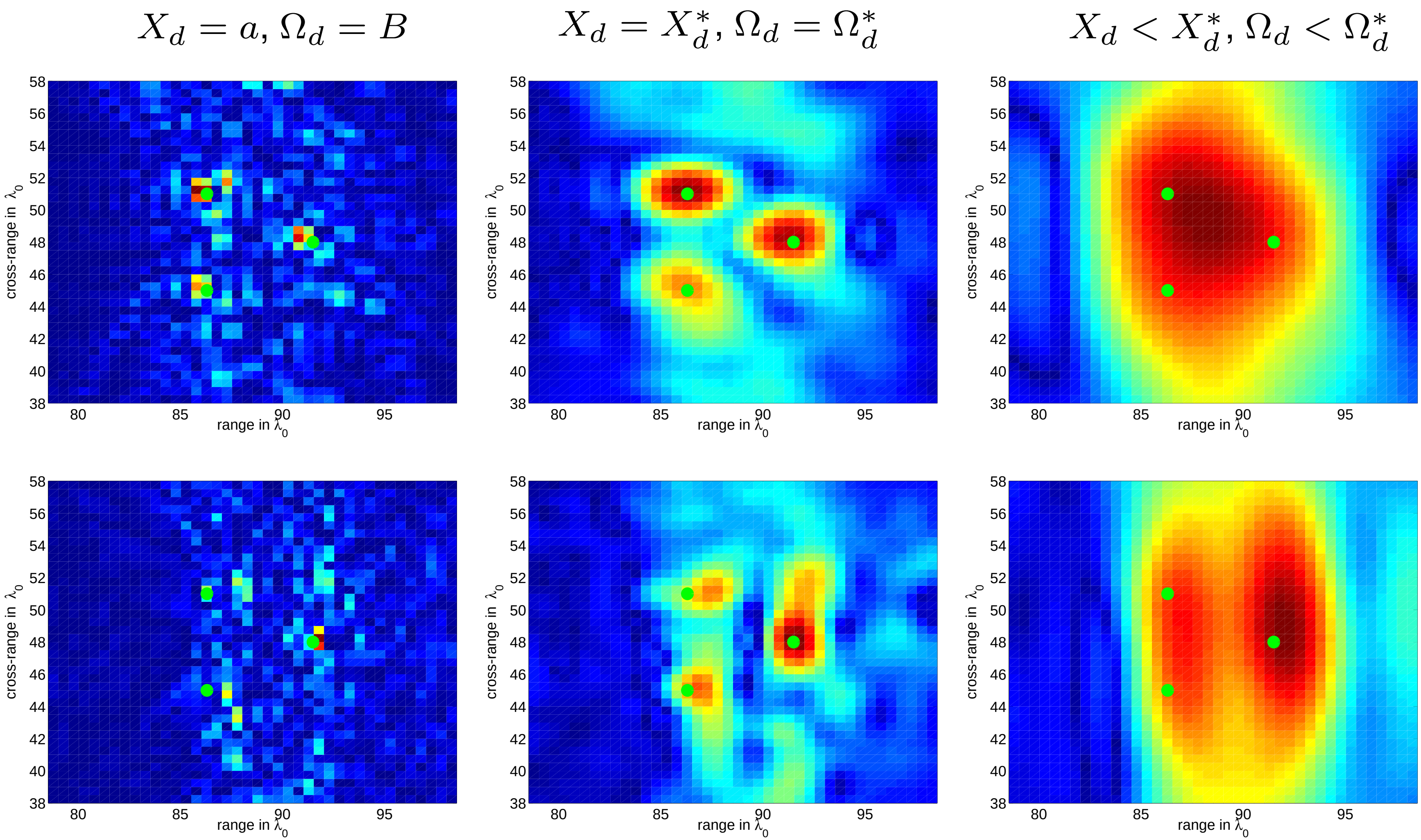
CINT results

Active array



Results for three different realizations of the clutter. Top: migration. Bottom: CINT

CINT results



Fixed clutter realization: effect of decoherence parameters on the image. Left: no smoothing. Middle: optimal smoothing. Right: too much smoothing.

Top row: passive array. Bottom row: active array.

Adaptive Selection of κ_d and Ω_d

- base the selection of κ_d, Ω_d on the image itself !
  - ✔ minimize the image support (sparse representation that reduces the blurring)
  - ✔ minimize rapid oscillations in the image.

Adaptive Selection of κ_d and Ω_d

- base the selection of κ_d, Ω_d on the image itself !
  - ✔ minimize the image support (sparse representation that reduces the blurring)
  - ✔ minimize rapid oscillations in the image.
- We minimize the objective functional

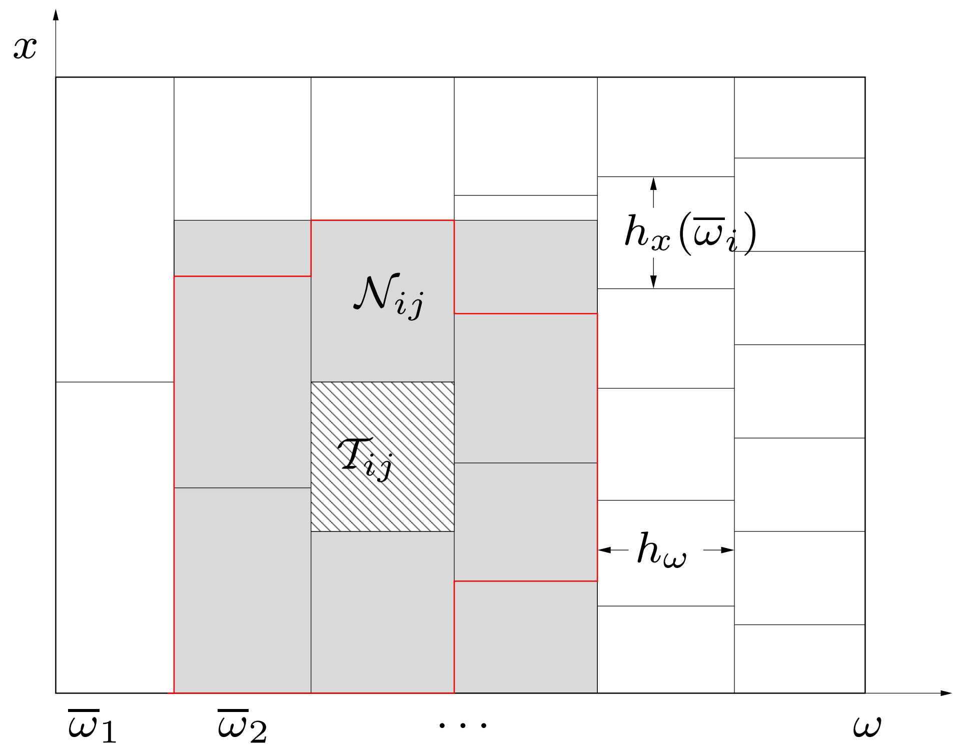
$$\mathcal{O}(\mathbf{y}^s; \Omega_d, \kappa_d) = \|\mathcal{J}_{\mathcal{N}}(\mathbf{y}^s; \Omega_d, \kappa_d)\|_{L^1(\mathcal{D})} + \alpha \, \|\nabla_{\mathbf{y}^s} \mathcal{J}_{\mathcal{N}}(\mathbf{y}^s; \Omega_d, \kappa_d)\|_{L^1(\mathcal{D})} \, ,$$

$$\mathcal{J}_{\mathcal{N}}(\mathbf{y}^s; \Omega_d, \kappa_d) = \sqrt{|\mathcal{I}^{\text{CINT}}(\mathbf{y}^s; \Omega_d, \kappa_d)|} / \sup_{\mathbf{y}^s \in \mathcal{D}_s} \sqrt{|\mathcal{I}^{\text{CINT}}(\mathbf{y}^s; \Omega_d, \kappa_d)|}$$

we use $\alpha = 1$

Implementation

- Introduce a tiling $(h_\omega, h_{\mathbf{x}}(\omega))$ of the $(\omega, \mathbf{x})$ plane,

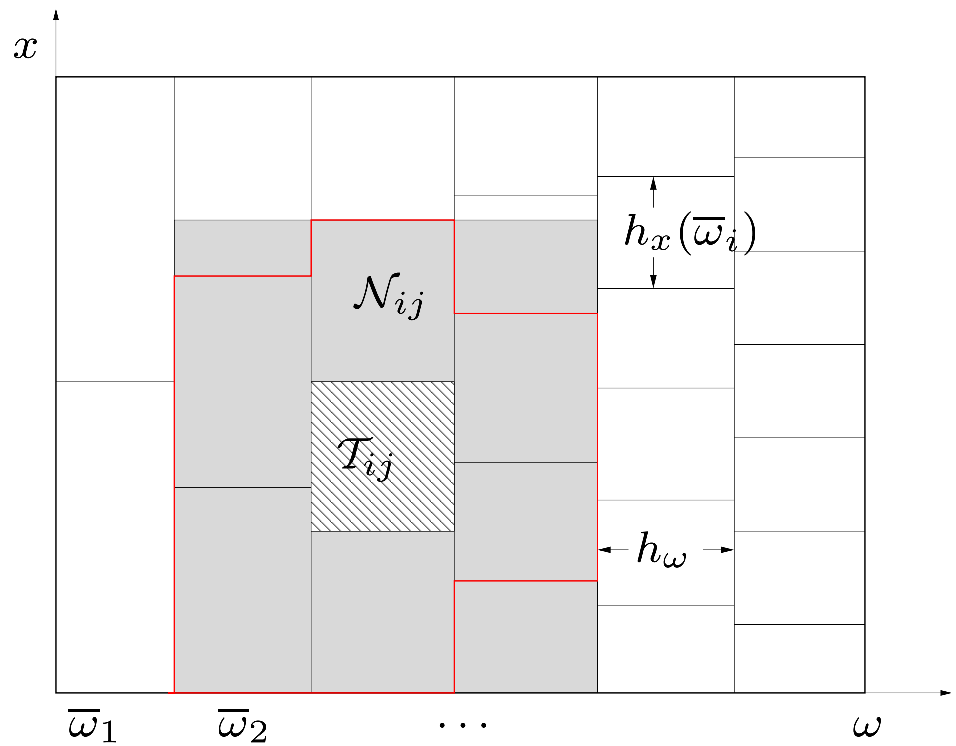


- we approximate $\mathcal{I}^{\text{CINT}}(\mathbf{y}^s)$ by,

$$\mathcal{J}(\mathbf{y}^s) = \sum_{(\mathbf{x}_r,\omega)\in a\times B} \sum_{(\mathbf{x}_r{}',\omega')\in\mathcal{N}(\omega,\mathbf{x}_r)} \hat{Q}(\mathbf{x}_r, \mathbf{x}_s, \omega; \mathbf{y}^S)\overline{\hat{Q}(\mathbf{x}_r{}', \mathbf{x}_s, \omega'; \mathbf{y}^S)}$$

Implementation

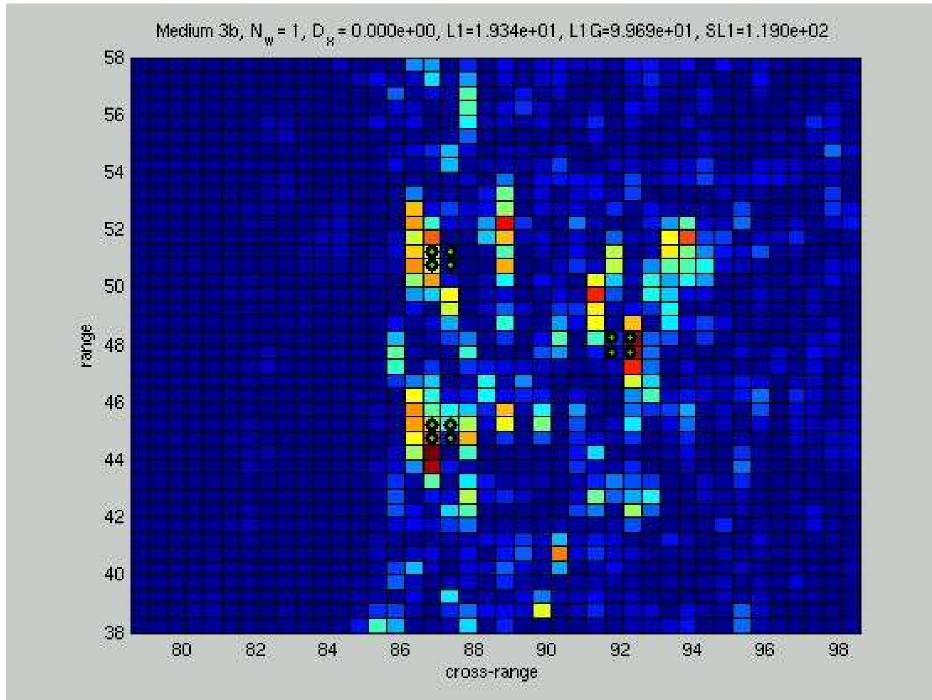
- Introduce a tiling $(h_\omega, h_{\mathbf{x}}(\omega))$ of the $(\omega, \mathbf{x})$ plane,



- For tile centered at $(\overline{\mathbf{x}}_j, \overline{\omega_i})$, we have

$\Omega_d = 2h_\omega$ and $X_d(\overline{\omega}_i) = 2h_x(\overline{\omega}_i) = \kappa_d^{-1} c_0/\overline{\omega}_i$

Adaptive CINT results



- We use the NOMADm software package (C. Audet, J. Dennis, M. Abramson), that uses a mesh-adaptive direct search method for constrained, nonlinear, mixed variable problems.
- For this example $\Omega_d^* = B/5$ and $\kappa_d^* = 0.125$.

Imaging resolution

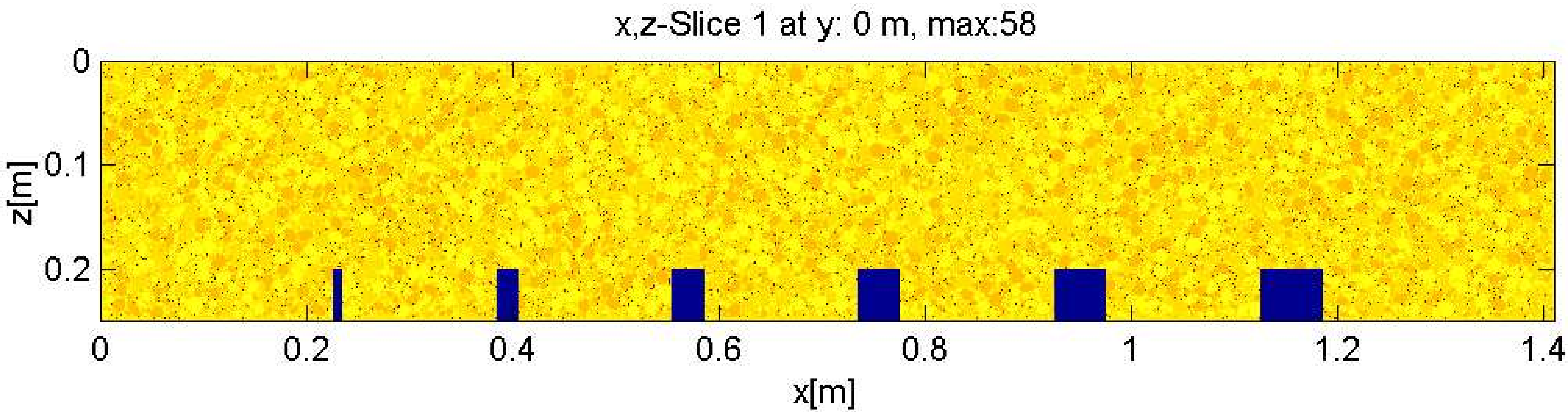
- migration resolution in homogeneous media
  - in range : $O\left(\frac{c_0}{B}\right)$
  - in cross-range : $O\left(\lambda\frac{L}{a}\right) = O\left(\frac{c_0 L}{\omega a}\right)$
- CINT resolution in clutter ($\Omega_d < B$ & $X_d < a$)
  - in range : $O\left(\frac{c_0}{\Omega_d}\right)$
  - in cross-range : $O\left(L\kappa_d\right) = O\left(\frac{c_0 L}{\overline{\omega} X_d(\overline{\omega})}\right)$
- for $\Omega_d \ll B$ & $X_d \ll a$
  - ✔ incoherent imaging should be used (diffusion)

$$D = \frac{c_0 \ell^*}{3}$$

  - CINT works for $L < \ell^*$ (in numerics $\ell^* = 75\lambda_0$)

Real data: measurements

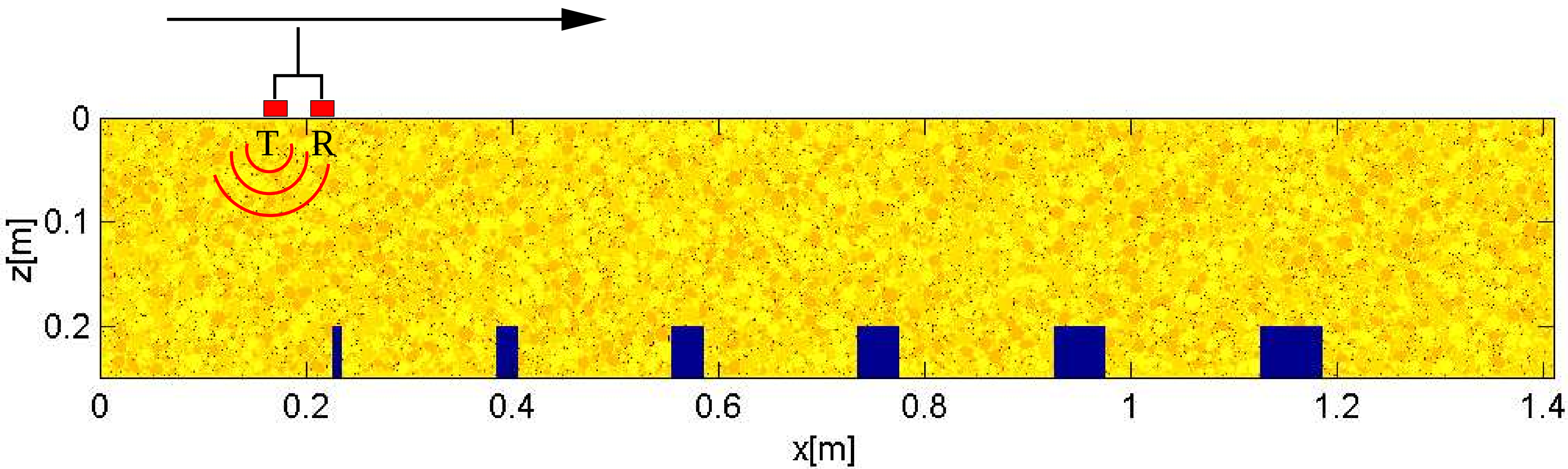
- Concrete structure to be imaged



- data provided by K. Mayer, University of Kassel, Germany.
- simulation in homog. medium: $f_0 = 200$KHz, $c_0 = 4207$m/s
- experimental data: $f_0 = 150$KHz, $c_L = 4150$m/s
- Transmitter and receiver: Krautgrämer G0,2R

Real data: measurements

- Measurement acquisition geometry



Real data: measurements

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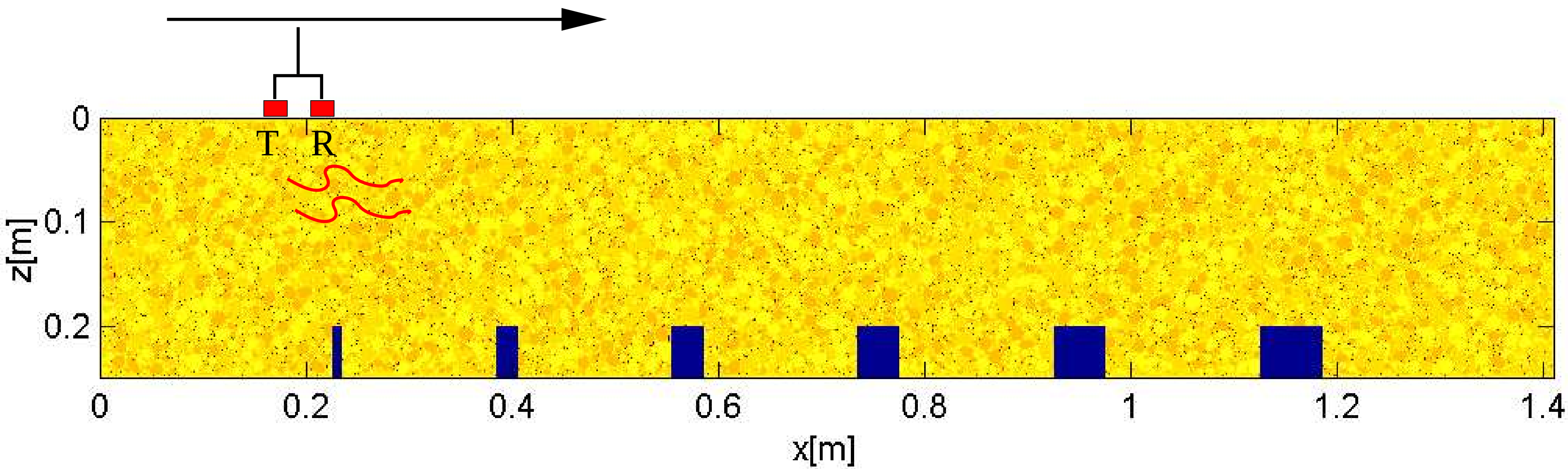
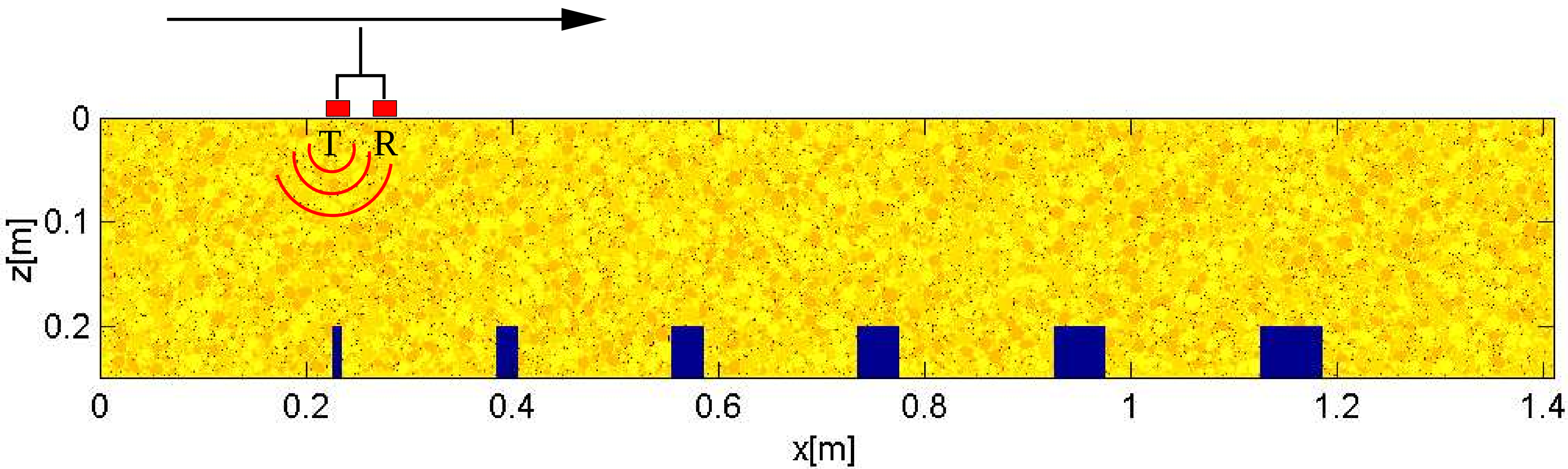


Real data: measurements

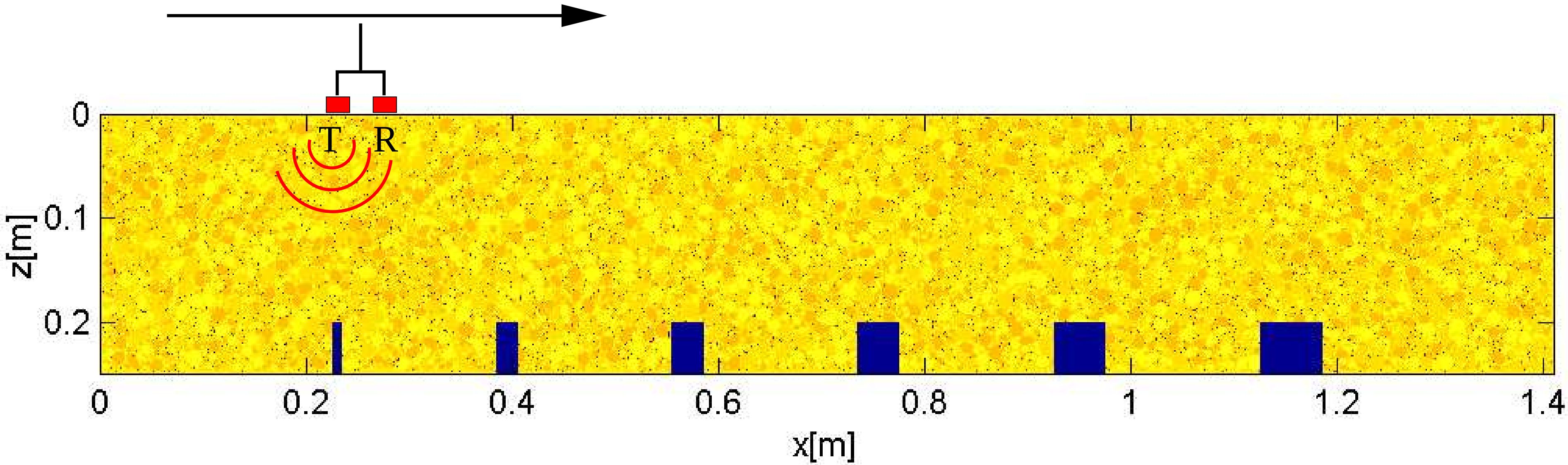
- Measurement acquisition geometry



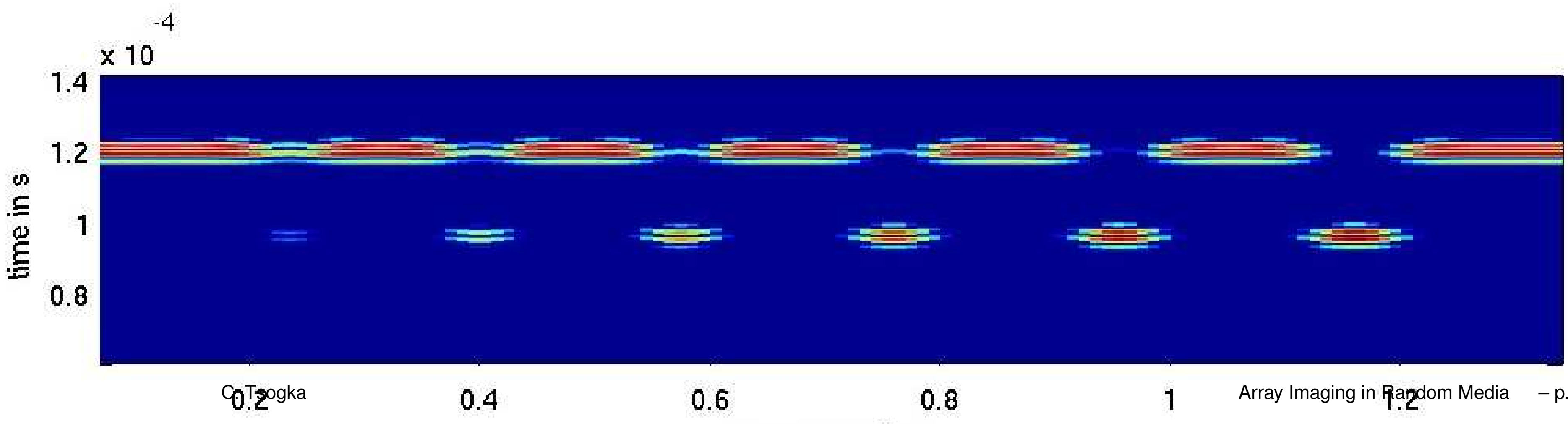
- There is no array here the aperture is synthetic as in SAR.

Real data: measurements

- Measurement acquisition geometry



- Simulated data traces in homogeneous structure



The CINT functional

- We rewrite the CINT imaging functional

$$\mathcal{I}^{\text{CINT}}(\mathbf{y}^S, \Omega_d, \kappa_d) = \int_B d\omega \int_{|\omega-\omega'|\leq\Omega_d} d\omega' \sum_{\mathbf{x}_m\in a} \sum_{|\mathbf{x}_m-\mathbf{x}_m'|\leq X_d(\omega)}$$

$$\hat{Q}(\mathbf{x}_m - \frac{d}{2}, \mathbf{x}_m + \frac{d}{2}, \omega, \mathbf{y}^s)\overline{\hat{Q}(\mathbf{x}_m' - \frac{d}{2}, \mathbf{x}_m' + \frac{d}{2}, \omega', \mathbf{y}^s)}$$

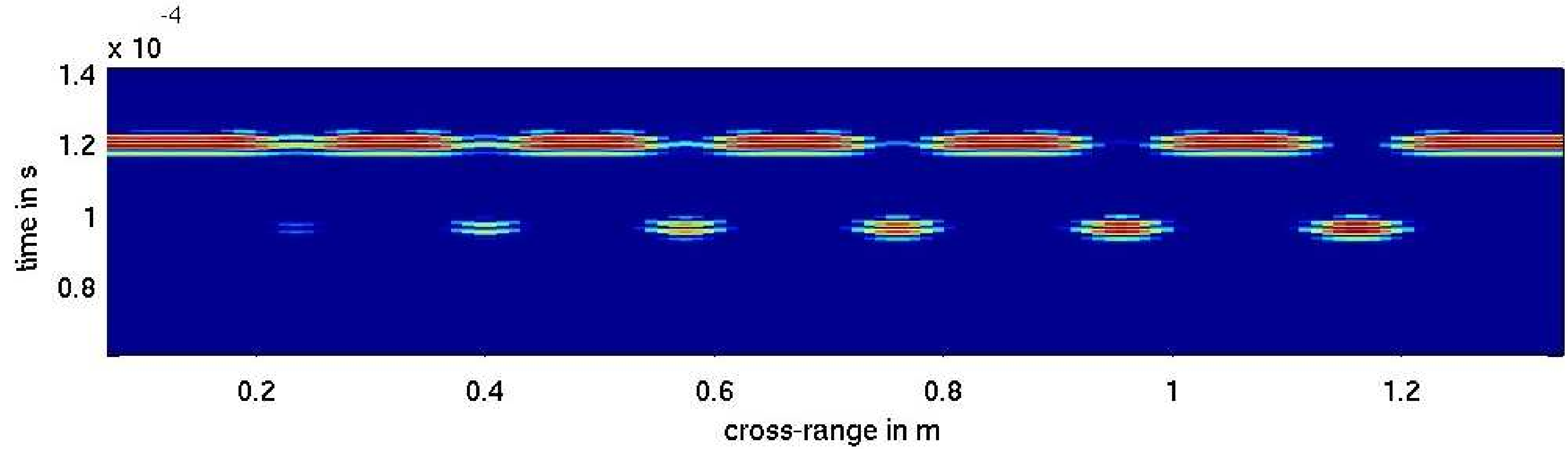
$$\hat{Q}(\mathbf{x}_s, \mathbf{x}_r, \omega, \mathbf{y}^S) = \hat{P}(\mathbf{x}_s, \mathbf{x}_r, \omega) e^{-i\omega\left(\tau(\mathbf{x}_s,\mathbf{y}^S)+\tau(\mathbf{x}_r,\mathbf{y}^S)\right)}$$

with

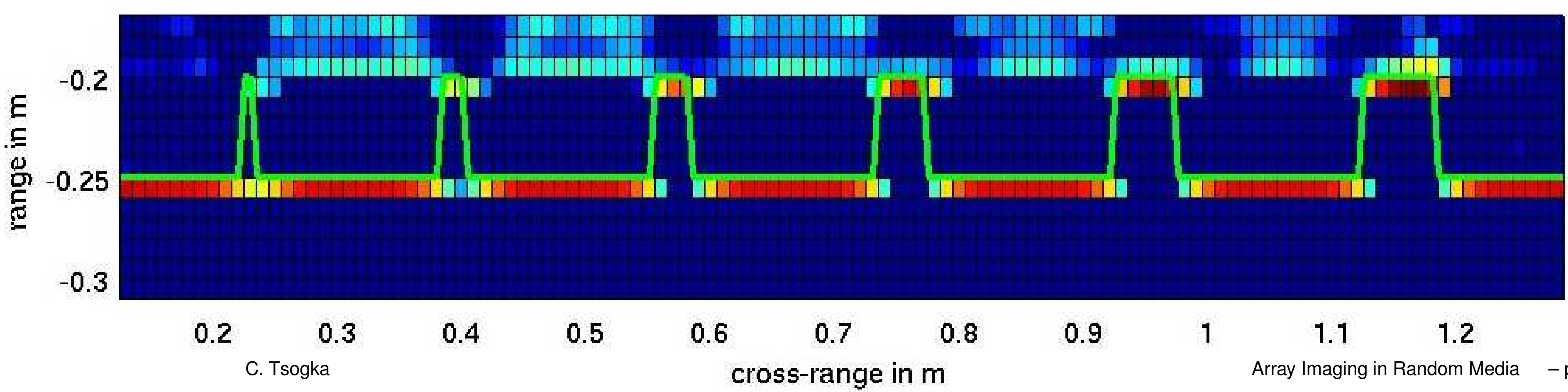
- $\mathbf{x}_m$: the midpoint moving on the array.
- d: distance between transmitter and receiver (fixed).
- $\mathbf{x}_s = \mathbf{x}_m - \frac{d}{2}, \mathbf{x}_r = \mathbf{x}_m + \frac{d}{2}$

Non-destructive testing results

- Simulated data traces in homogeneous structure

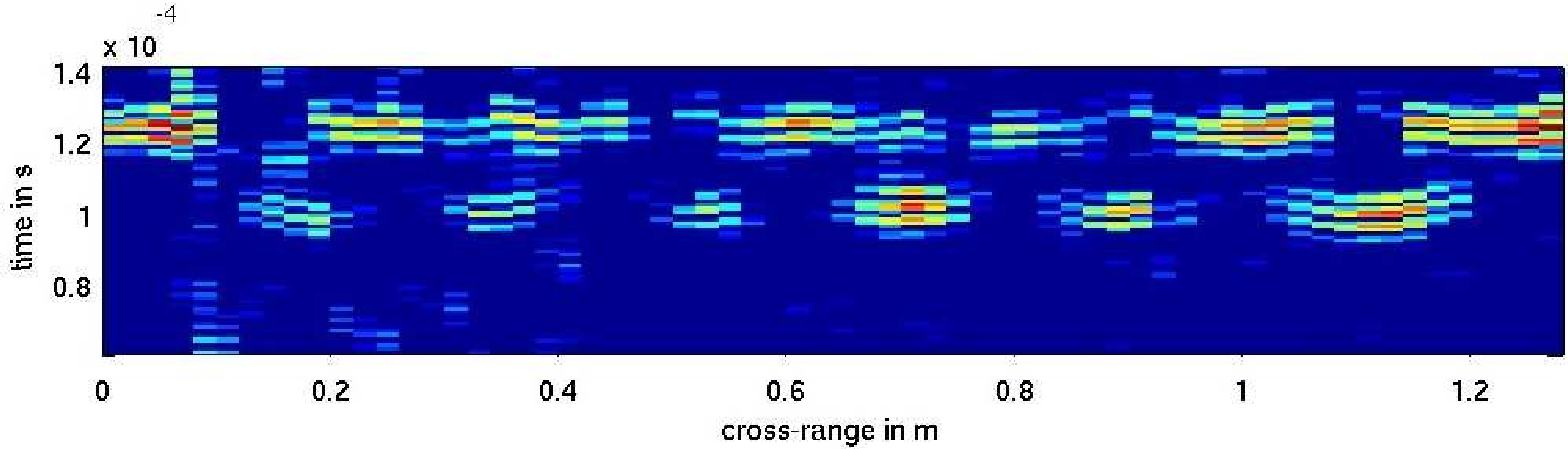


- Kirchhoff migration results



Non-destructive testing results

- Data traces in the concrete structure



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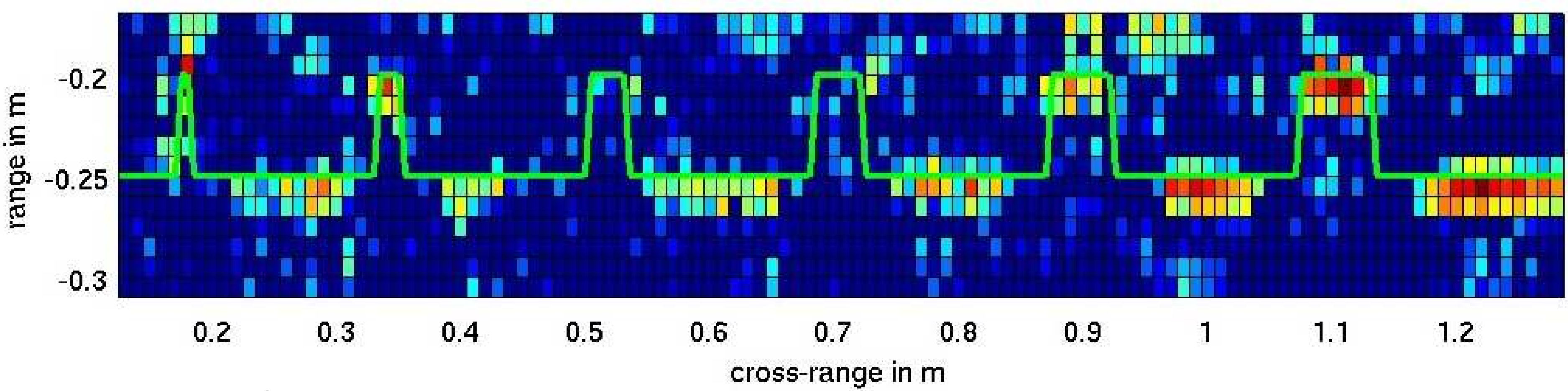
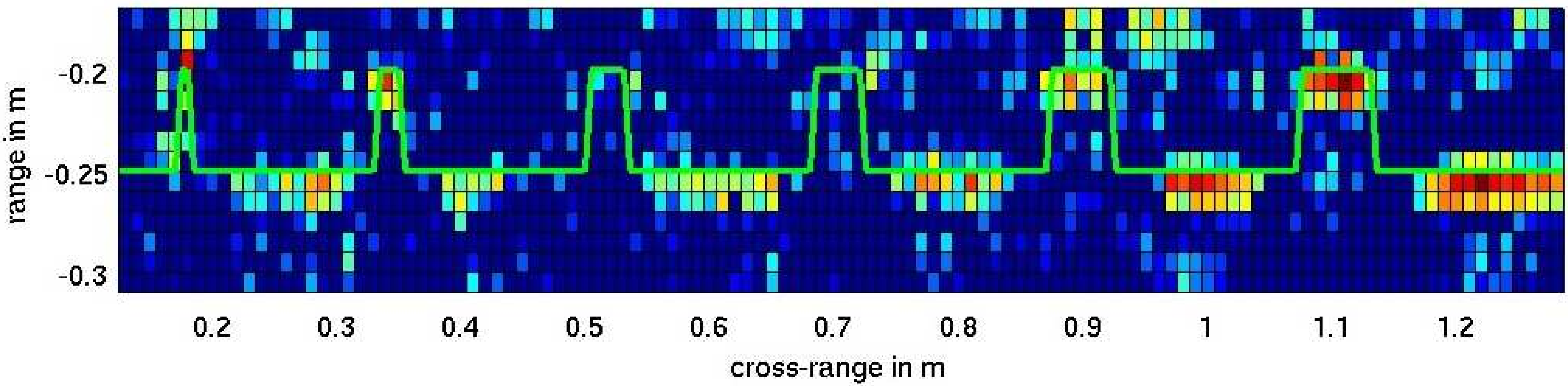


Non-destructive testing results

- Kirchhoff migration results



- CINT results