CAAM 570

Homework 3

Due at the beginning of class on Feb 27. No late submissions accepted.

**8.1.3** Given a graph $G(x, y)$, consider the problem of deciding whether $G$ has an $xy$-path of odd (respectively, even) length.

i.e., given a graph G and two vertices x,y in G, consider the problem of deciding whether G has a path subgraph with odd (even) length and endpoints x and y.

a) Show that this problem:
   i) belongs to $\mathcal{NP}$,
   ii) belongs to co-$\mathcal{NP}$.

b) Describe a polynomial-time algorithm for solving the problem.

**⋆8.3.5**

For the following questions, only assume the problem DIRECTED HAMILTON CYCLE is NP-Complete

a) Describe a polynomial reduction of DIRECTED HAMILTON CYCLE to HAMILTON CYCLE.

b) Deduce that HAMILTON CYCLE $\in \mathcal{NPC}$.

**8.3.6** Let HAMILTON PATH denote the problem of deciding whether a given graph has a Hamilton path.

a) Describe a polynomial reduction of HAMILTON CYCLE to HAMILTON PATH.

b) Deduce that HAMILTON PATH $\in \mathcal{NPC}$.

**8.3.7** Two problems $P$ and $Q$ are *polynomially equivalent*, written $P \equiv Q$, if $P \preceq Q$ and $Q \preceq P$.

a) Show that:

$$\text{HAMILTON PATH} \equiv \text{HAMILTON CYCLE} \equiv \text{DIRECTED HAMILTON CYCLE}$$

b) Let MAX PATH denote the problem of finding the length of a longest path in a given graph. Show that MAX PATH $\equiv$ HAMILTON PATH.

**8.3.8**

a) Let $k$ be a fixed positive integer. Describe a polynomial-time algorithm for deciding whether a given graph has a path of length $k$.

b) The length of a longest path in a graph $G$ can be determined by checking, for each $k$, $1 \leq k \leq n$, whether $G$ has a path of length $k$. Does your algorithm for the problem in part (a) lead to a polynomial-time algorithm for MAX PATH?