CAAM 570  
Homework 3  
Due at the beginning of class on Feb 27. No late submissions accepted.

8.1.3 Given a graph \( G(x, y) \), consider the problem of deciding whether \( G \) has an \( xy \)-path of odd (respectively, even) length.

a) Show that this problem:
   i) belongs to \( \mathcal{NP} \),
   ii) belongs to \( \text{co-} \mathcal{NP} \).

b) Describe a polynomial-time algorithm for solving the problem.

\*8.3.5 For the following questions, only assume the problem \textsc{Directed Hamilton Cycle} is \text{NP-Complete}

a) Describe a polynomial reduction of \textsc{Directed Hamilton Cycle} to \textsc{Hamilton Cycle}.

b) Deduce that \textsc{Hamilton Cycle} \( \in \mathcal{NP} \).

8.3.6 Let \textsc{Hamilton Path} denote the problem of deciding whether a given graph has a Hamilton path.

a) Describe a polynomial reduction of \textsc{Hamilton Cycle} to \textsc{Hamilton Path}.

b) Deduce that \textsc{Hamilton Path} \( \in \mathcal{NP} \).

8.3.7 Two problems \( P \) and \( Q \) are \textit{polynomially equivalent}, written \( P \equiv Q \), if \( P \leq Q \) and \( Q \leq P \).

a) Show that:

\[ \text{Hamilton Path} \equiv \text{Hamilton Cycle} \equiv \text{Directed Hamilton Cycle} \]

b) Let \textsc{Max Path} denote the problem of finding the length of a longest path in a given graph. Show that \( \text{Max Path} \equiv \text{Hamilton Path} \).

8.3.8

a) Let \( k \) be a fixed positive integer. Describe a polynomial-time algorithm for deciding whether a given graph has a path of length \( k \).

b) The length of a longest path in a graph \( G \) can be determined by checking, for each \( k, 1 \leq k \leq n \), whether \( G \) has a path of length \( k \). Does your algorithm for the problem in part (a) lead to a polynomial-time algorithm for \textsc{Max Path}?