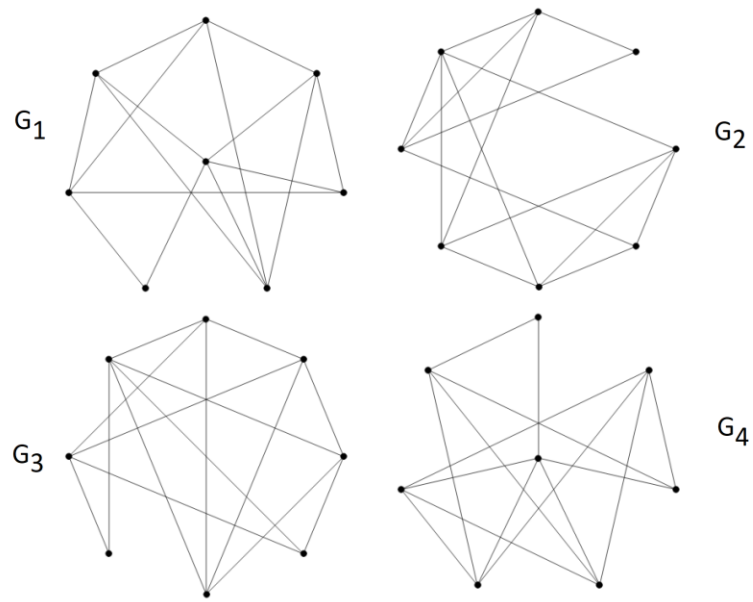
CAAM 570
Midterm exam

- **You may use up to 5 hours for completing the midterm.**
- **If you want, after you're done working, you may neatly rewrite or typeset parts of your exam that look illegible due to bad handwriting; this won't count toward your 5 hour limit, as long as you don't change what you've written before.**
- **The exam must be completed and turned in by the beginning of class on March 8, 2018. You can hand it in during class, or leave it in my mailbox in Duncan Hall, or in my office.**
- **You may use your class notes, homeworks, and the textbook by Bondy&Murty.**
- **Do not use any other sources, including the internet.**
- **Write the honors pledge below affirming that you have followed these directions.**
- **Don't forget to (legibly) write your name on whatever you turn in.**

(Q1 – 15 pts)

Which, if any, of the following graphs are isomorphic? Justify your answer.



(Q2 – 20 pts: 5, 15)

A graph G is self-complementary if it is isomorphic to its complement.

a) Draw all self-complementary graphs on ≤ 5 vertices.
b) Prove that a self-complementary graph has diameter ≤ 3.

(Q3 – 20 pts: 15, 5)

a) Show that every simple graph G with $\delta(G) \geq 2$ contains a path of length at least $\delta(G)$ and a cycle of length at least $\delta(G) + 1$.
b) For each $k \geq 2$, find a graph G with $\delta(G) = k$ which contains no path of length greater than k, and a graph G with $\delta(G) = k$ which contains no cycle of length greater than $k + 1$.

(Q4 – 30 pts: 5, 5, 10, 10)

Give a polynomial time algorithm (or a closed-form expression) for

a) finding the minimum cut of a tree;
b) finding the maximum cut of a tree;
c) finding a Hamilton cycle of a grid graph $P_{n_1} \square P_{n_2}$, or determining that one doesn't exist;
d) finding the maximum independent set of a k-dimensional grid graph $P_{n_1} \square P_{n_2} \square \ldots \square P_{n_k}$.

Justify your answers.

(Q5 – 15 pts)

Prove that the following problem is NP-Complete, or that it can be solved in polynomial time (or, for $1,000,000, prove both).

Instance: A graph $G = (V, E)$
Question: Does G contain a clique of order exactly $|V|/2$?