Worksheet #8: Spectral in time

(1) Consider the initial boundary value problem (IBVP)

\[
\begin{align*}
\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} &= 0 & 0 < x < 50 \\
u(x,0) &= 5 - \frac{1}{8}|x - 25| \\
u(0,t) &= u(50,t) = 0 & \text{for } t > 0.
\end{align*}
\]

(a) Plot \(u(x,0)\).

(b) Derive the solution to the IBVP via the spectral method.

(c) Plot the solution at \(t = 0, 0.02, 0.04\) and \(0.06\). Also plot the steady state solution.

(2) Consider the initial boundary value problem (IBVP)

\[
\begin{align*}
\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} &= x & 0 < x < 50 \\
u(x,0) &= 5 - \frac{1}{8}|x - 25| \\
u(0,t) &= u(50,t) = 0 & \text{for } t > 0.
\end{align*}
\]

(a) Derive the solution to the IBVP via the spectral method.

(b) Plot the solution at \(t = 0, 0.02, 0.04\) and \(0.06\). Also plot the steady state solution.

(3) Consider the initial boundary value problem (IBVP)

\[
\begin{align*}
\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} &= 1/2 - x & 0 < x < 1, \ t > 0 \\
u(x,0) &= x(1 - x) \\
\frac{\partial u}{\partial x}(0,t) + \frac{\partial u}{\partial x}(1,t) &= 0 & \text{for } t > 0.
\end{align*}
\]

(a) Derive the solution to the IBVP via the spectral method.

(b) Plot the solution at \(t = 0, 0.02, 0.04\) and \(0.06\). Also plot the steady state solution.