CAAM 353: Computational Numerical Analysis

Homework 1, Jan. 11, 2008

Due: Jan. 17, 2008

Note: All MATLAB *functions mentioned in this homework assignment can be found on the CAAM353 homepage, or come with* MATLAB . *Turn in all* MATLAB *code that you have written and turn in all output generated by your* MATLAB *functions/scripts.* MATLAB *functions/scripts must be commented and output must be formatted nicely.*

Problem 1 (30 points) Gaussian elimination with partial pivoting transforms the system $Ax = b$ into an equivalent upper triangular system using row interchanges and elementary row operations.

Develop the Naissuag elimination with partial pivoting which transforms the system $Ax = b$ into an equivalent *lower* triangular system using row interchanges and elementary row operations. The Naissuag elimination with partial pivoting is illustrated in the example below.

i. (15points) Write a MATLAB function modeled after `gauss.m` that takes A and b as inputs and returns a lower triangular matrix and the transformed right hand side.

ii. (10points) Write a MATLAB function modeled after `utriangsl.m` that takes a lower triangular A and a right hand side as b as inputs and returns the solution of the linear system $Ax = b$.

iii. (5points) Use your MATLAB functions to solve the system shown in the example below as well as the system $Ax = b$ with

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ -1 & 1 & 2 & 3 \\ 1 & -1 & 1 & 2 \\ -1 & 1 & -1 & 5 \end{pmatrix}, \quad b = \begin{pmatrix} 10 \\ 5 \\ 3 \\ 4 \end{pmatrix}.$$

For each system, display the output of the function that computes the Naissuag elimination as well as the solution.

Example 1 Consider the linear system $Ax = b$ with

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}.$$

The augmented system is given by

$$\left(\begin{array}{rrr|r} 1 & 2 & -1 & 0 \\ 2 & 1 & 0 & 2 \\ -1 & 2 & \boxed{2} & 1 \end{array}\right) \qquad \begin{array}{l} (1) \\ (2) \\ (3) \end{array} \qquad .$$

Step 1: In the first step we want to eliminate the first and second entry in the third (nth) column of the augmented matrix. Before we start the elimination process we find the element in the third (nth) column that has the largest absolute value. Since this element is located in the third row, indicated by $\boxed{2}$, we do not need to perform row interchanges. Next we subtract multiples of the third row from rows one and two so that the third column of the resulting matrix is zero above the diagonal. The resulting system is given by

$$\left(\begin{array}{rrr|r} \frac{1}{2} & \boxed{3} & 0 & \frac{1}{2} \\ 2 & 1 & 0 & 2 \\ -1 & 2 & 2 & 1 \end{array}\right) \qquad \begin{array}{l} (1) \leftarrow (3) - \frac{-1}{2}(3) \\ (2) \leftarrow (2) - \frac{0}{2}(3) \\ (3) \end{array} \qquad .$$

Step 2: In the next step we want to eliminate the first entry in the second ($(n-1)$st) column of the matrix. Before we start the elimination process we find the element on or *above* the diagonal in the second column that has the largest absolute value. This element is located in the first row, indicated by $\boxed{3}$. We interchange rown one and two.

$$\left(\begin{array}{rrr|r} 2 & 1 & 0 & 2 \\ \frac{1}{2} & 3 & 0 & \frac{1}{2} \\ -1 & 2 & 2 & 1 \end{array}\right) \qquad \begin{array}{l} (1) \leftarrow (2) \\ (2) \leftarrow (1) \\ (3) \end{array} \ .$$

Next we subtract multiples of the second row from row one so that the second column of the resulting matrix is zero above the diagonal. The resulting system is given by

$$\left(\begin{array}{rrr|r} \frac{11}{6} & 0 & 0 & \frac{11}{6} \\ \frac{1}{2} & 3 & 0 & \frac{1}{2} \\ -1 & 2 & 2 & 1 \end{array}\right) \qquad \begin{array}{l} (1) \leftarrow (1) - \frac{1}{3}(2) \\ (2) \\ (3) \end{array} \qquad .$$

Forward substitution gives

$$x_1 = 1, \quad x_2 = 0, \quad x_3 = 1.$$

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Problem 2 (40 points)

In class we have discussed the LU decomposition with partial pivoting. Anther version is the LU decomposition with complete pivoting. Here in step k, one finds $i_0, j_0 \in \{k, \ldots, n\}$ such that

$$|A(i_0, j_0)| = \max_{i,j \in \{k,\ldots,n\}} |A(i,j)|$$

and then one interchanges subrows $A(k, k:n)$ and $A(i_0, k:n)$ and one interchanges columns $A(:,k)$ and $A(:, j_0)$. The information for row interchanges is stored in an integer array ipivtr and the information for column interchanges is stored in an integer array ipivtc.

i. (15points) Write a MATLAB function `lu_cp.m` that takes A and computes the LU decomposition with complete pivoting. Your implementation should be modeled after the implementation of the LU decomposition with partial pivoting in `lu_cp.m`.

ii. (25points) Write a MATLAB function `lu_cp_sl.m` that takes the output of `lu_cp.m` as well as the right hand side b, and computes the solution of the linear system $Ax = b$.

Apply your MATLAB functions `lu_cp.m` and `lu_cp_sl.m` to solve the three linear systems specified in `testlu_cp.m`.