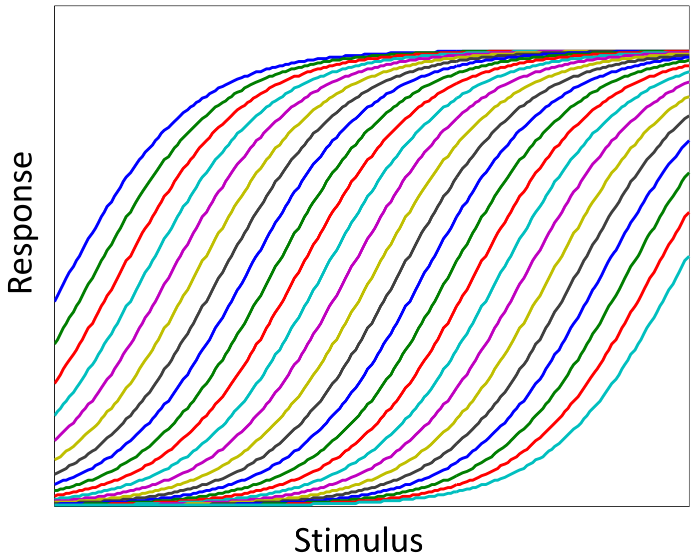
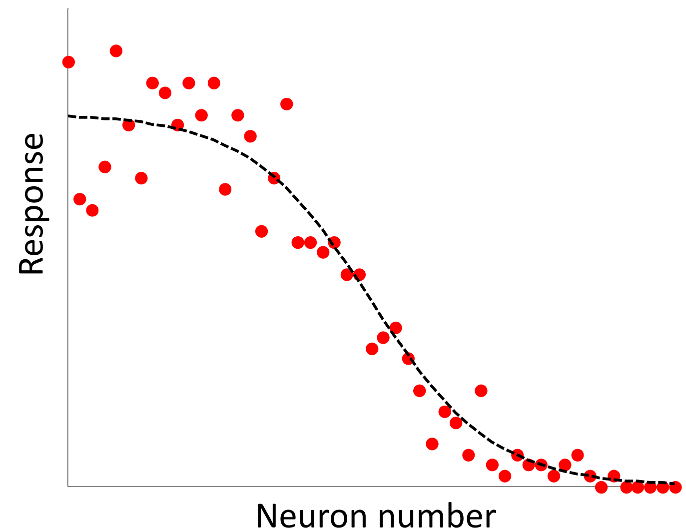
Last week’s homework

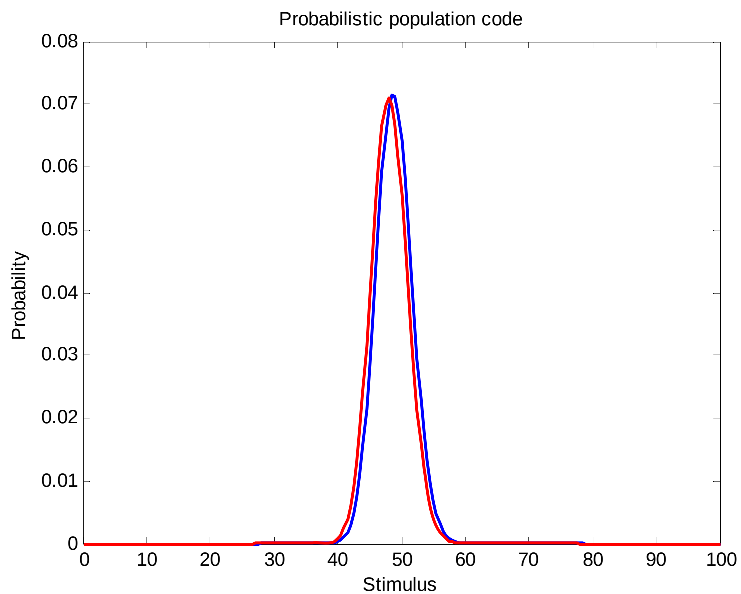
TUNING CURVES



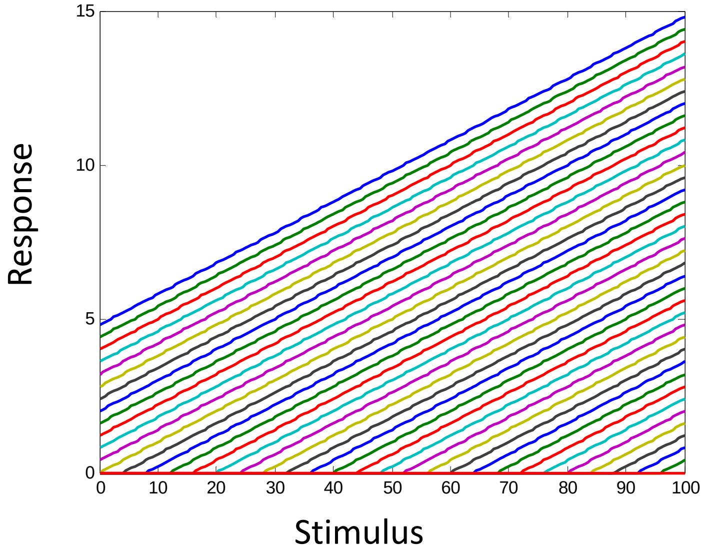
SINGLE-TRIAL ACTIVITY



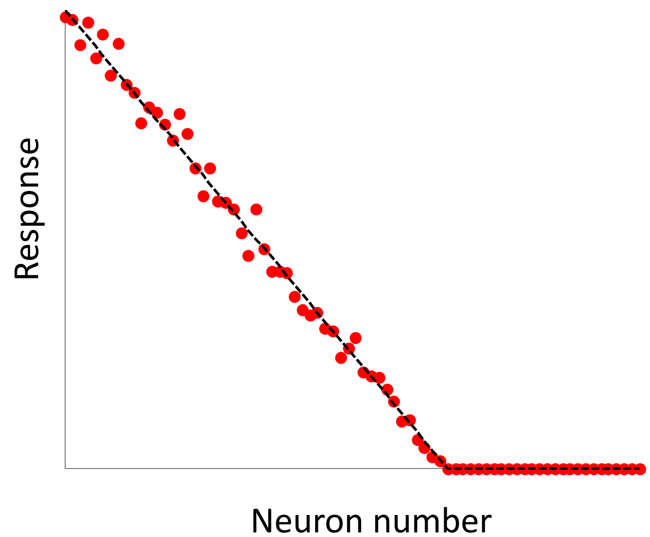
POSTERIOR DISTRIBUTION



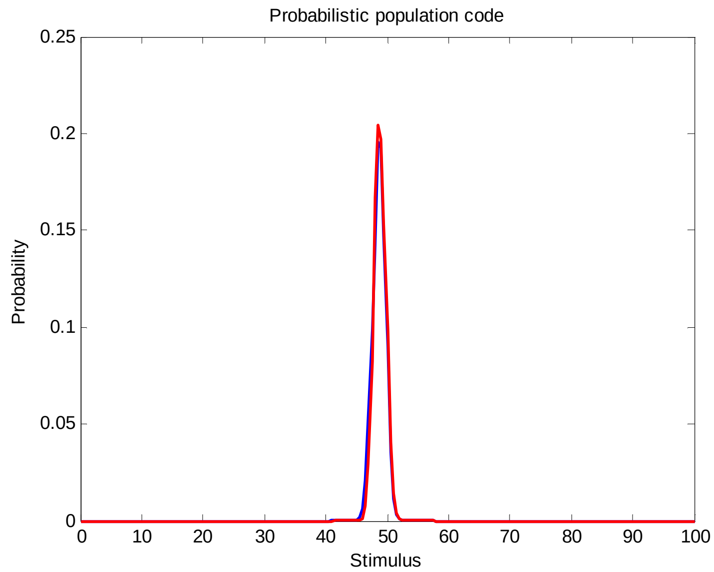
TUNING CURVES



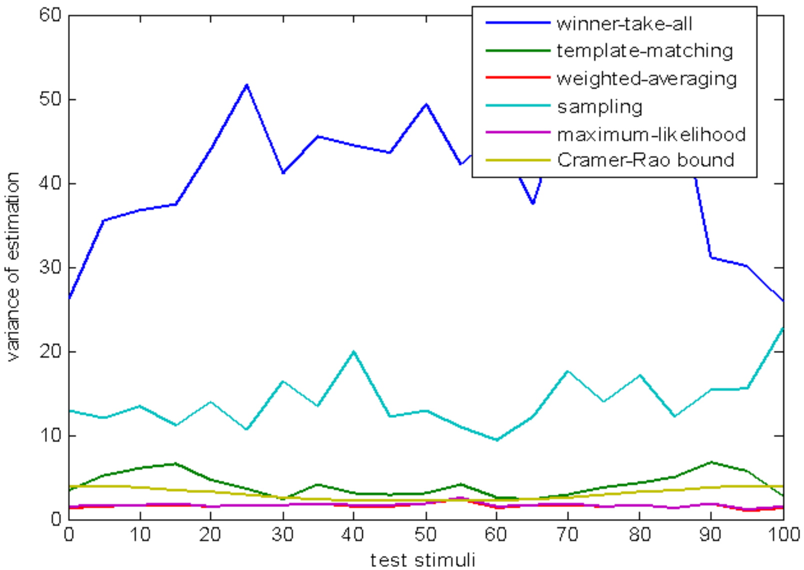
SINGLE-TRIAL ACTIVITY



POSTERIOR DISTRIBUTION

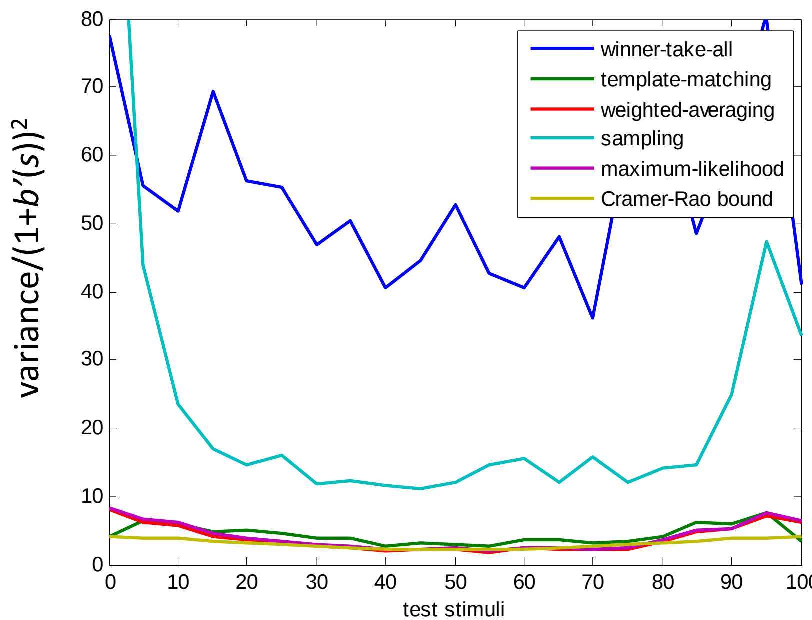


Student X

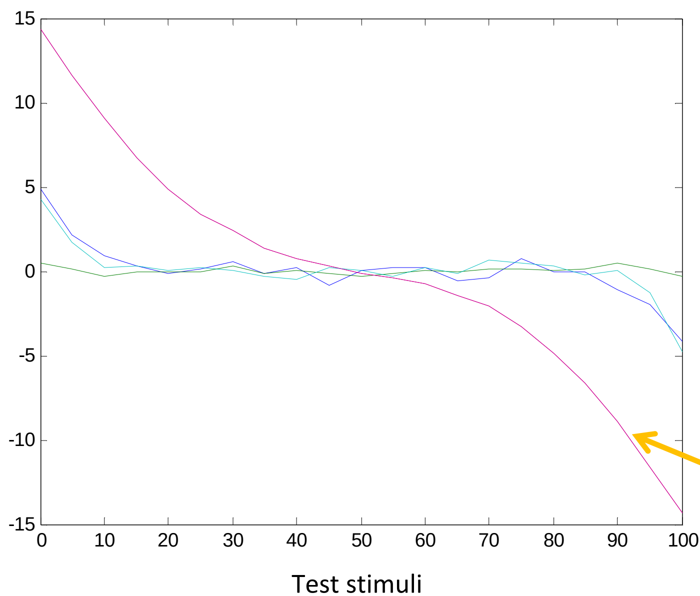


$$\frac{\sigma^2_{\text{any decoder}}(s)}{\left(1+b'_{\text{that decoder}}(s)\right)^2} \geq \frac{1}{I(s)}$$

Bias-corrected



Bias



Where does this bias come from?

First two lectures

- Coding a stimulus in a population of neurons
  - Population response distributions
  - Decoding methods and how to evaluate them
  - Effect of correlations on information content and decoding performance
- Limitations
  - Perceptual system is more than individual populations
  - Simplified stimuli (a single feature, or white noise)
  - All *representation*. What about *computation*?

Theoretical systems neuroscience

- … explains how neural representations of both simple and complex stimuli arise;
- … formalizes computation on stimuli at behavioral level;
- … explains how computations are implemented at neural level;
- … identifies neurons and neuronal circuits that actually implement these computations.

Marr's levels of analysis

- Computational: the problems vision must overcome
- Algorithmic: the strategy that is used
- Implementational: how it is done through neural operations

Analogy: cash register:

- Rules of addition
- Number system, operations on symbols
- Machinery of physical device

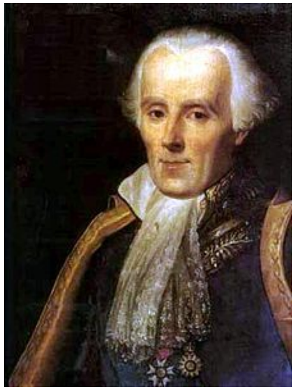
Example

- Computational: achieve high precision in the presence of noise
- Algorithmic: maximum-likelihood decoder
- Implementational: attractor dynamics

Probabilistic inference: complex perceptual computations

Al Hazen (Ibn al-Haytham), ca. 1030

“Perception requires unnoticed judgments.”

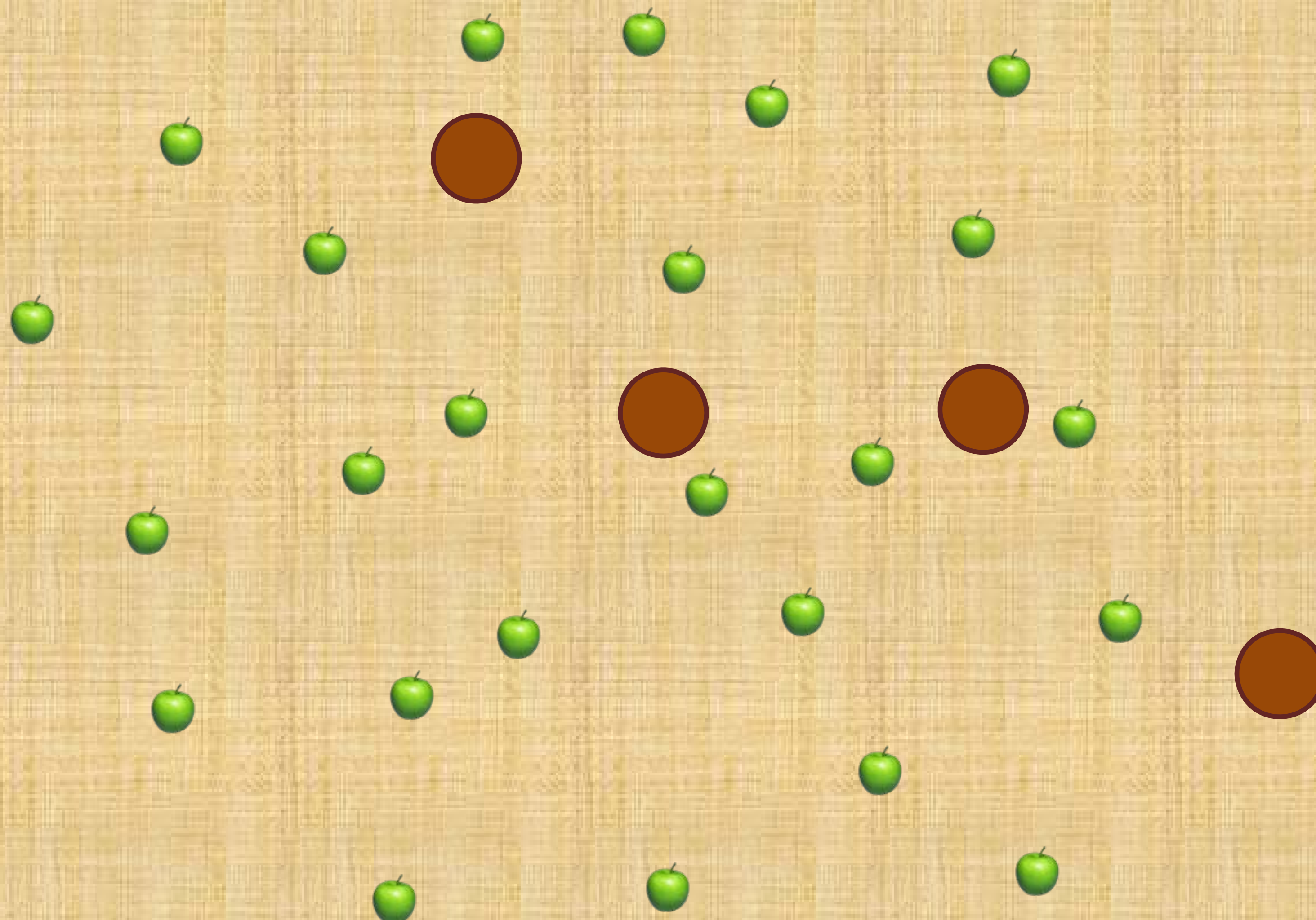


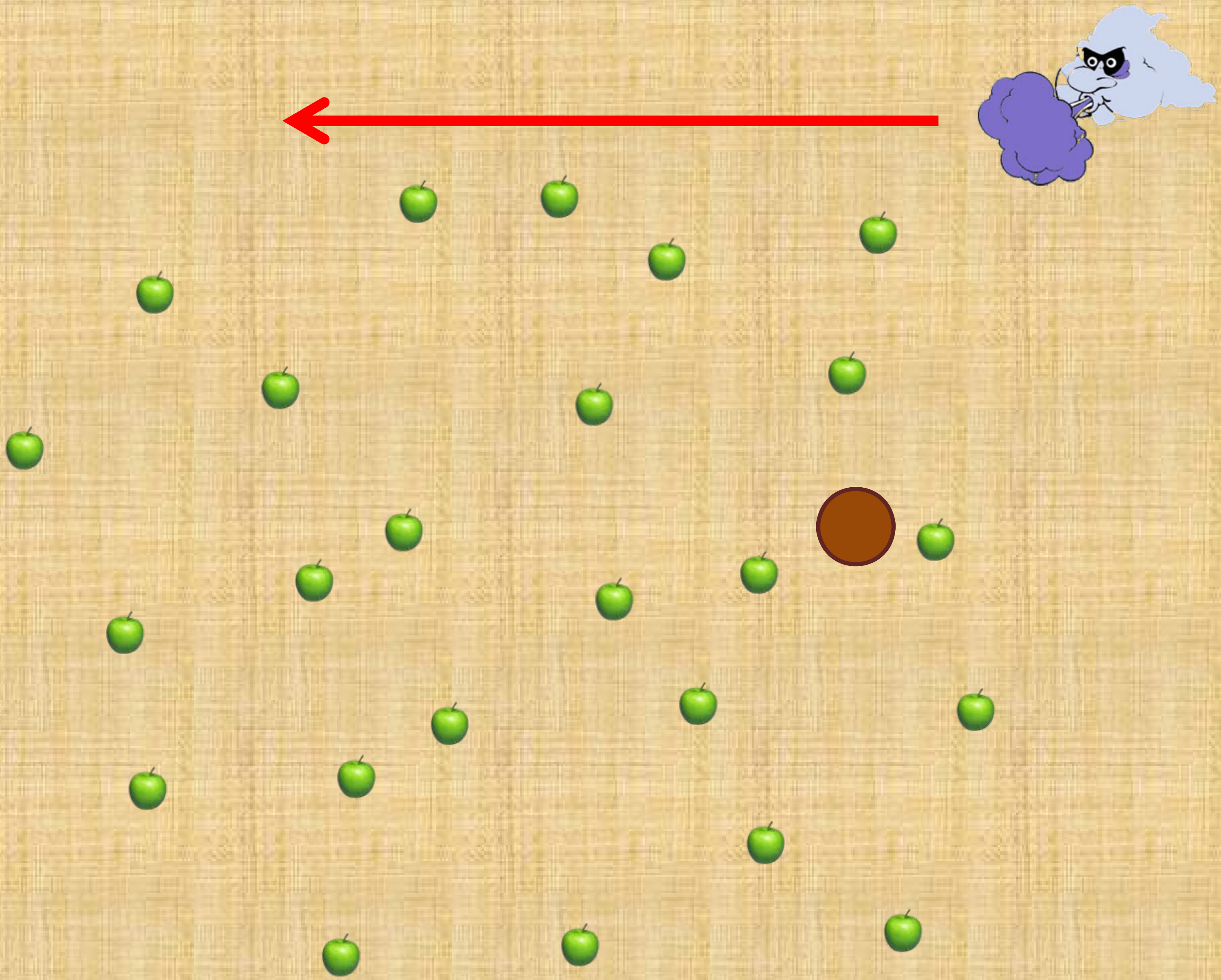
Pierre-Simon Laplace, 1825

“One may even say, strictly speaking, that almost all our knowledge is only probable; and in the small number of things that we are able to know with certainty, the principal means of arriving at the truth – induction and analogy – are based on probabilities.”

Hermann von Helmholtz, 1867

“Perception as unconscious inference”

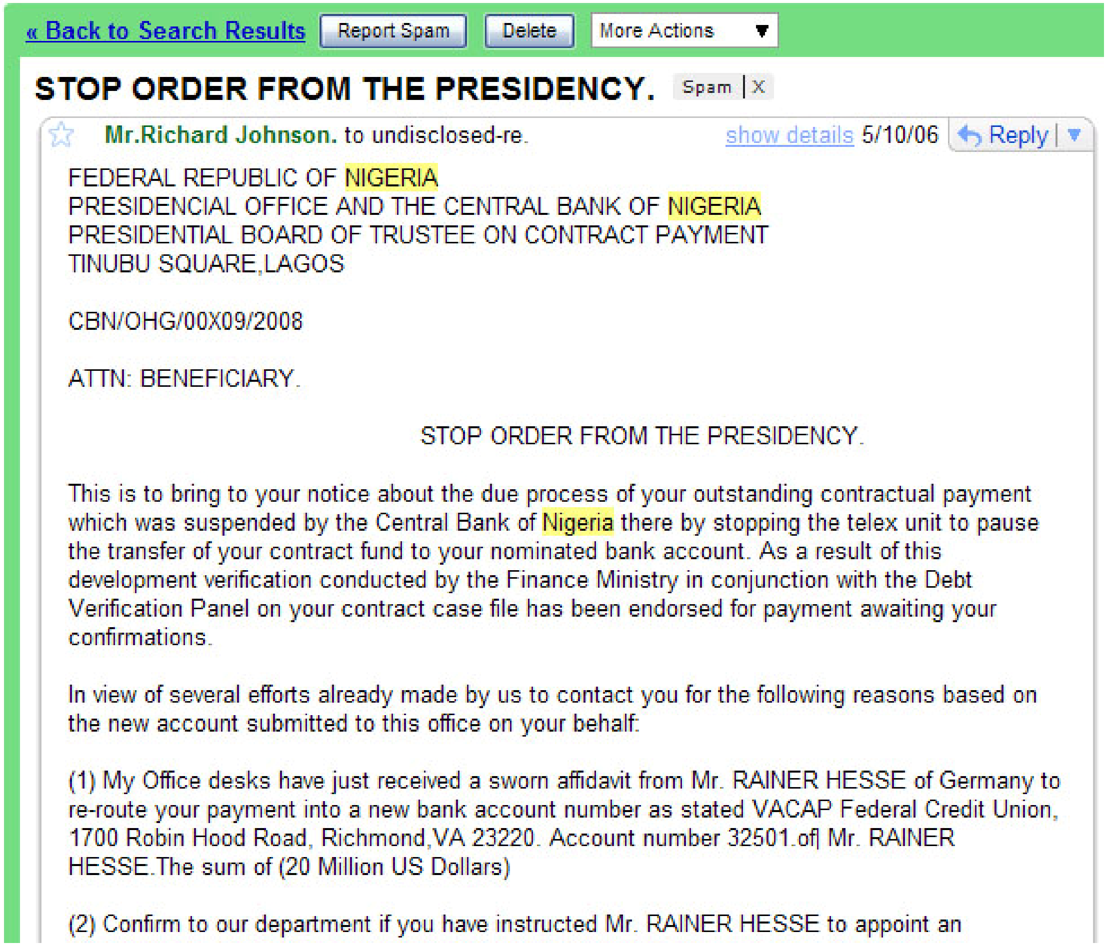




- Given noisy data, we can obtain the probability of possible events/causes …
- … as long as we know the process by which data are generated from a given event/cause.
- Bayes' rule:

$$p(\text{tree location} \mid \text{apples}) \propto p(\text{apples} \mid \text{tree location})$$

$$p(\text{event} \mid \text{data}) \propto p(\text{data} \mid \text{event})\, p(\text{event})$$



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STOP ORDER FROM THE PRESIDENCY. Spam | X

Mr.Richard Johnson. to undisclosed-re. show details 5/10/06 Reply

FEDERAL REPUBLIC OF NIGERIA
PRESIDENCIAL OFFICE AND THE CENTRAL BANK OF NIGERIA
PRESIDENTIAL BOARD OF TRUSTEE ON CONTRACT PAYMENT
TINUBU SQUARE,LAGOS

CBN/OHG/00X09/2008

ATTN: BENEFICIARY.

STOP ORDER FROM THE PRESIDENCY.

This is to bring to your notice about the due process of your outstanding contractual payment which was suspended by the Central Bank of Nigeria there by stopping the telex unit to pause the transfer of your contract fund to your nominated bank account. As a result of this development verification conducted by the Finance Ministry in conjunction with the Debt Verification Panel on your contract case file has been endorsed for payment awaiting your confirmations.

In view of several efforts already made by us to contact you for the following reasons based on the new account submitted to this office on your behalf:

(1) My Office desks have just received a sworn affidavit from Mr. RAINER HESSE of Germany to re-route your payment into a new bank account number as stated VACAP Federal Credit Union, 1700 Robin Hood Road, Richmond,VA 23220. Account number 32501.of| Mr. RAINER HESSE.The sum of (20 Million US Dollars)

(2) Confirm to our department if you have instructed Mr. RAINER HESSE to appoint an

$$p(\text{spam} \mid \text{words}) \propto p(\text{words} \mid \text{spam})\, p(\text{spam})$$

Bayesian inference in perception and cognition

$$p\left(\text{stimulus} \middle| \text{observations}\right)$$
$$\propto p\left(\text{observations} \middle| \text{stimulus}\right) p\left(\text{stimulus}\right)$$

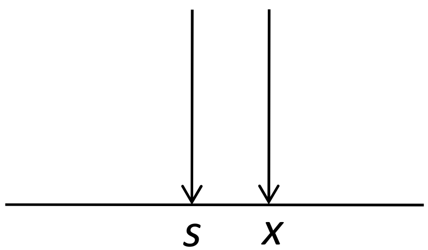
What is the stimulus ?
What are the observations?

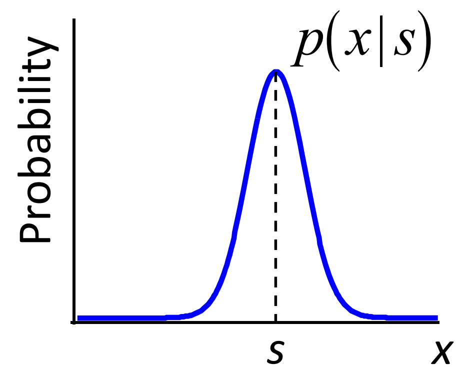
The stimulus (*s*)

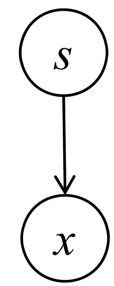
- Orientation of a line segment
- Color of an object
- Spatial location of an event or object
- Direction of motion of an object
- Identity of an object
- Cause of an odor
- Presence of a target in a scene
- Whether a change occurred between two scenes
- Identity of a spoken word
- Emotional state of a person
- Credibility of a person
- Who are my friends
- …

The observations

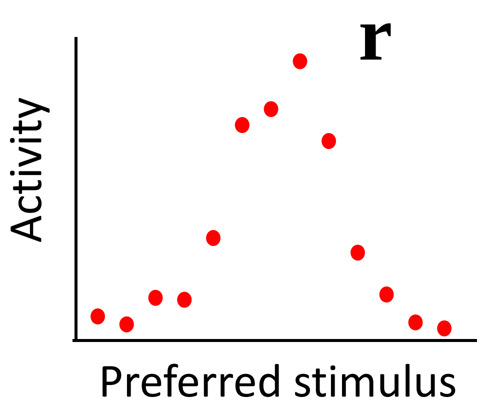
- "Internal representation" of the stimulus

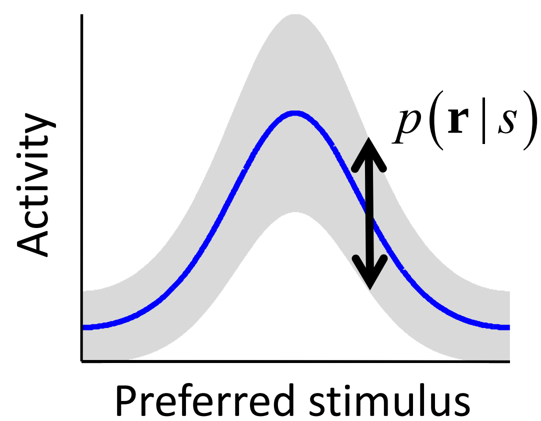


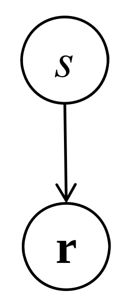




- Or: activity of a neural population (**r**)







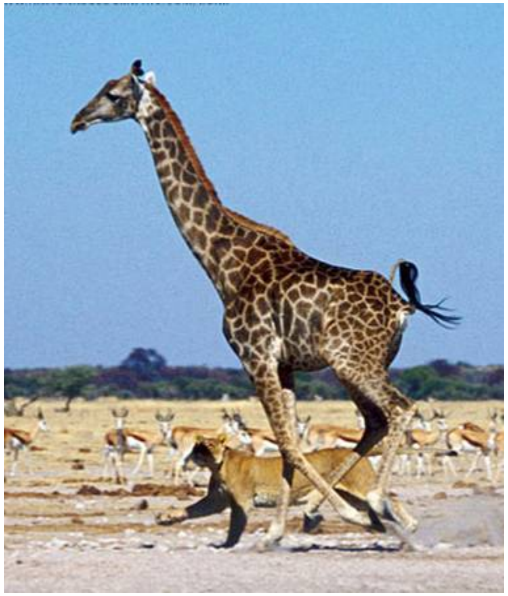
Inference

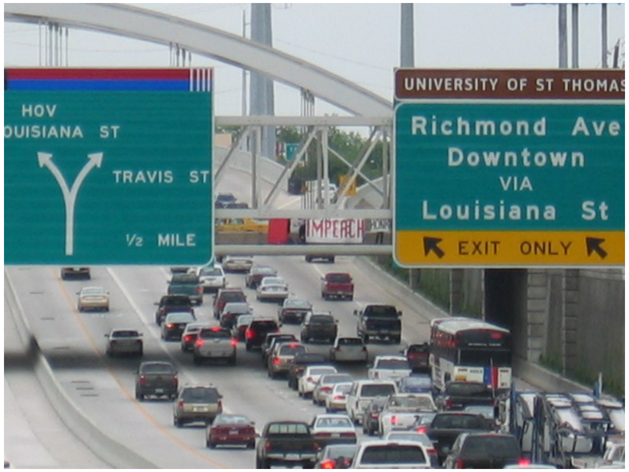
- Bayesian inference is the process of probabilistically inferring the stimulus from the observation(s):

$$p(s \mid x) \propto p(x \mid s)\, p(s)$$

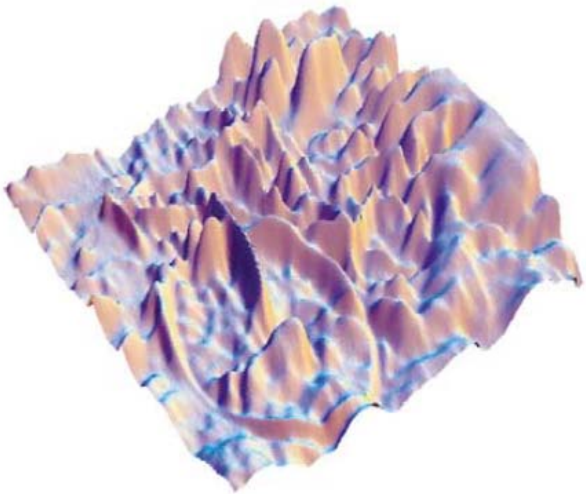
- *x* can be a complex collection of data
- *s* can be a completely different quantity than *x*
- An experimenter knows *s*, but not *x*;
  the observer's brain knows *x*, but not *s*
- Inferring *s* requires knowledge of generative model

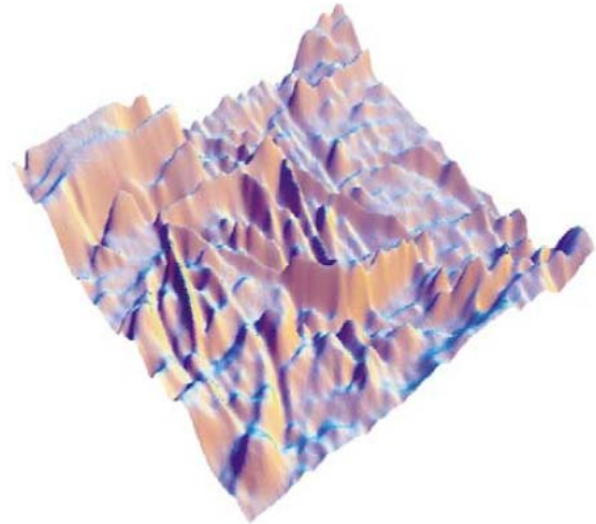
Inference in daily life

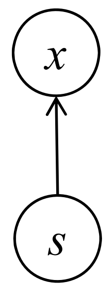




Object recognition

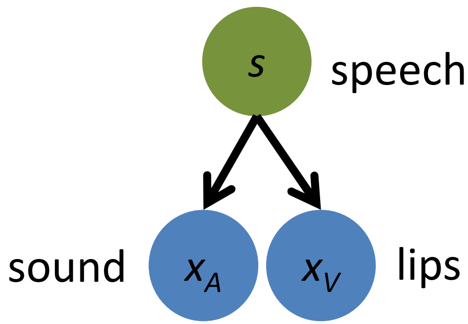




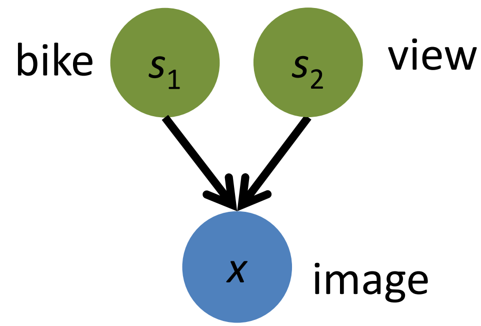


Kersten and Yuille, 2003

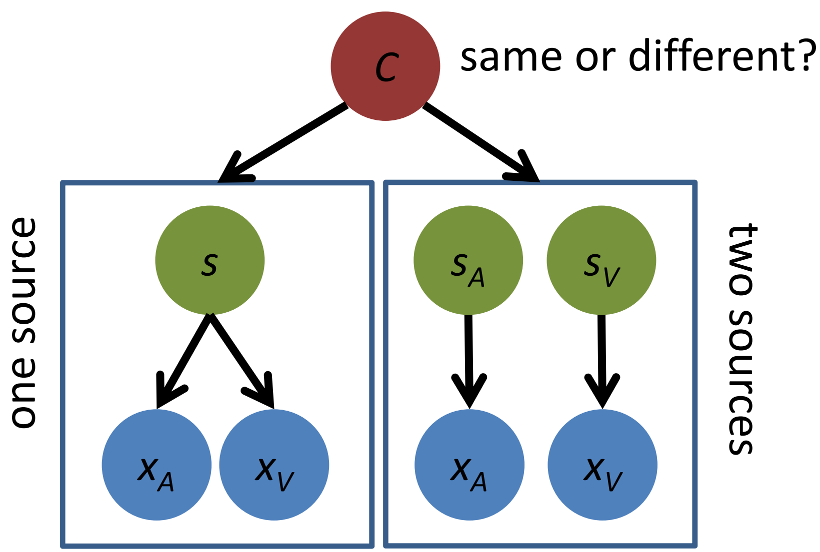
Generative models in perception



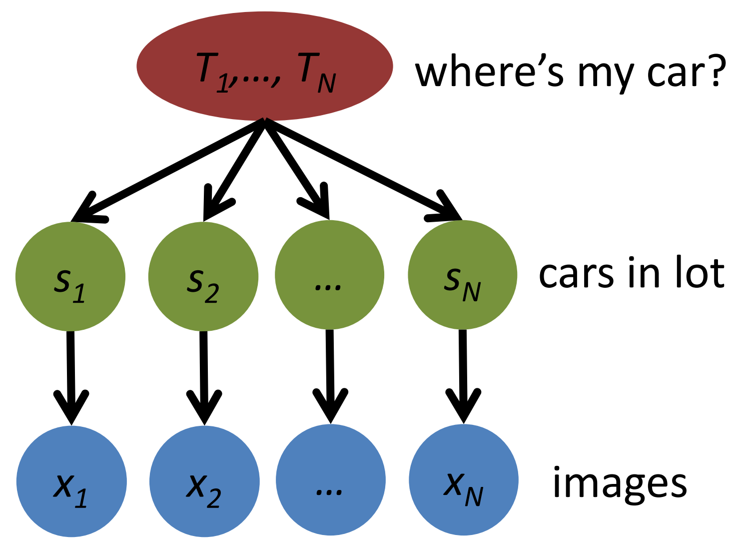
Cue integration



Invariant perception (discounting)



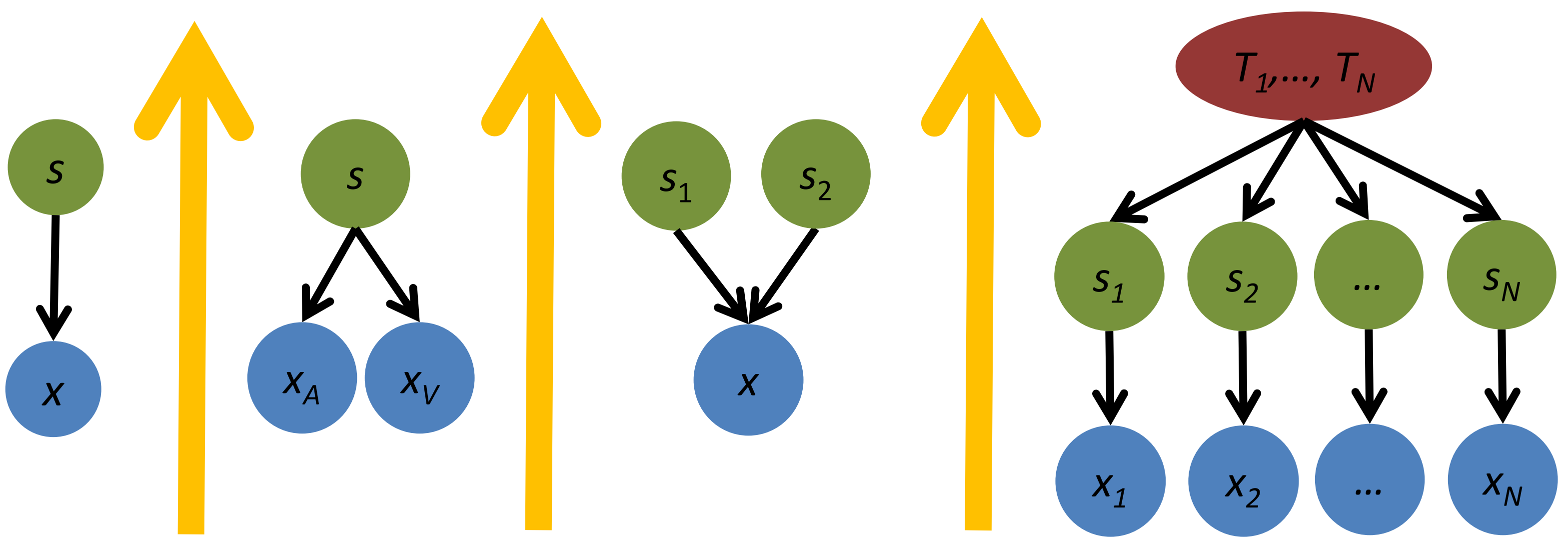
Causal inference



Visual search

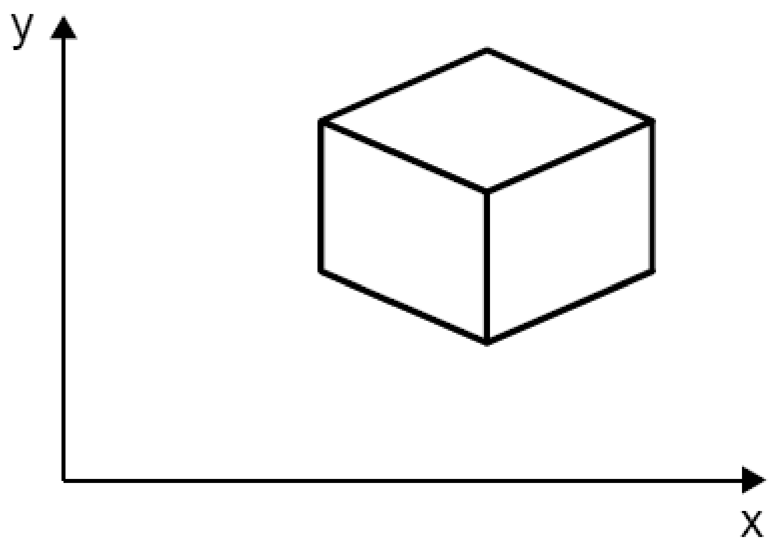
Inference

- Computing a probability distribution over a top-level variable (stimulus) based on the lowest-level variables (observations)



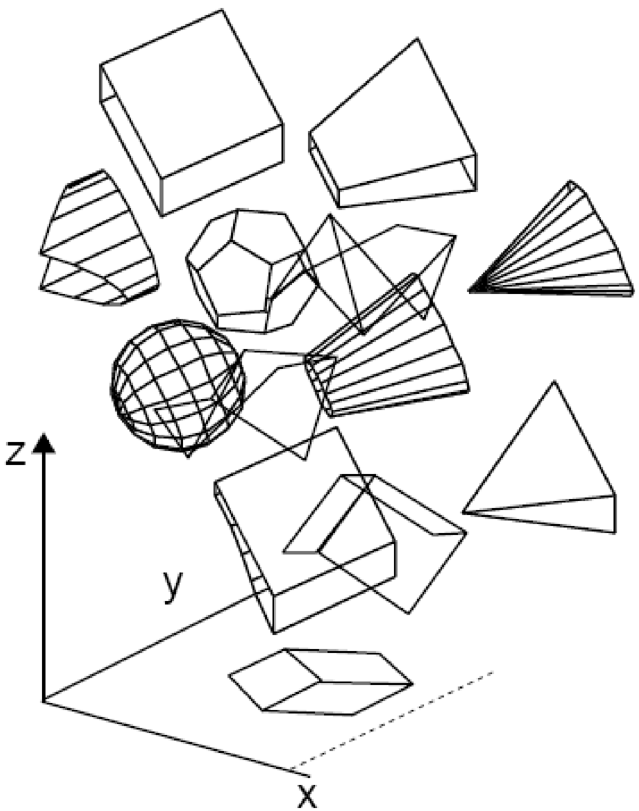
Object recognition

Image data *I*

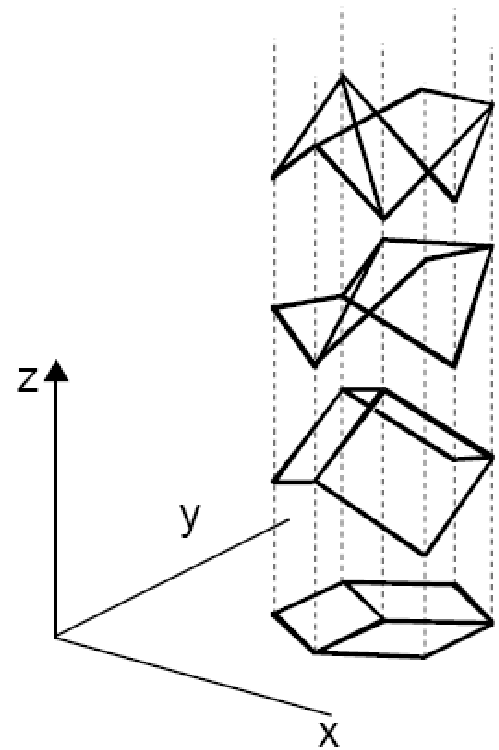


Kersten and Yuille, 2003

Prior over objects
$p(s)$



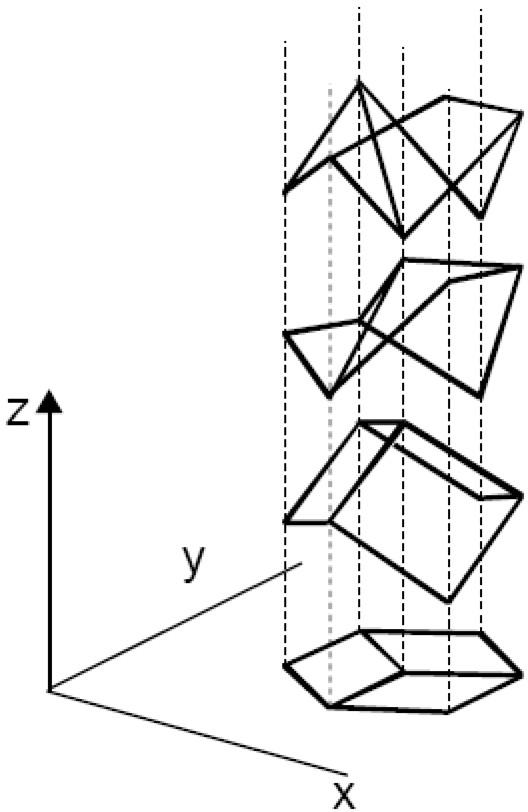
Likelihood over objects given 2D image
$p(I|s)$



Kersten and Yuille, 2003

Posterior over objects

$p(s|I)$



$p(s_1|I) = p_1$

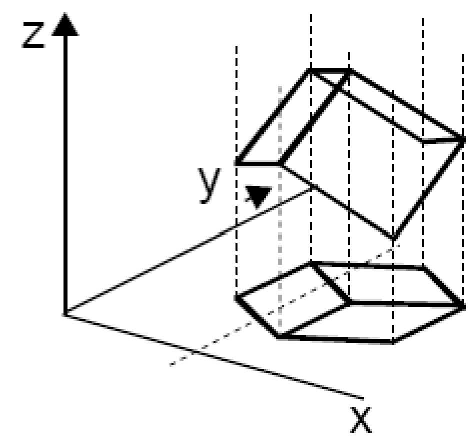
$p(s_2|I) = p_2$

$p(s_3|I) = p_3$

MAP estimate

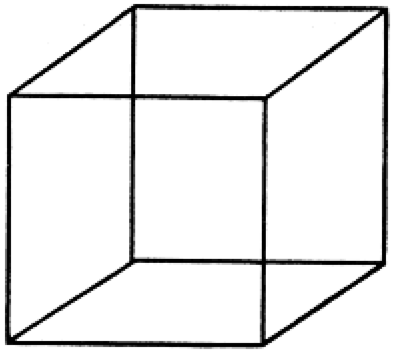
$$\hat{s} = \underset{s}{\operatorname{argmax}} \; p\left(s \mid I\right)$$

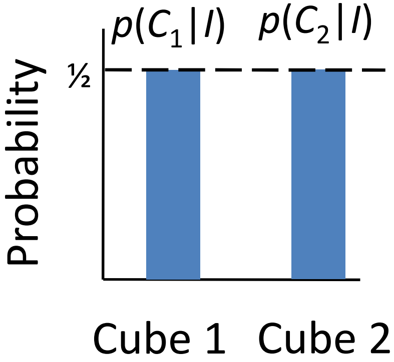
$p(s_3|I) = p_3$ is biggest



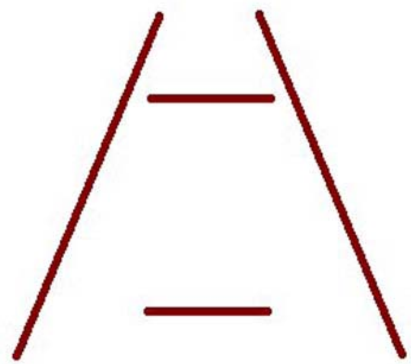
Kersten and Yuille, 2003

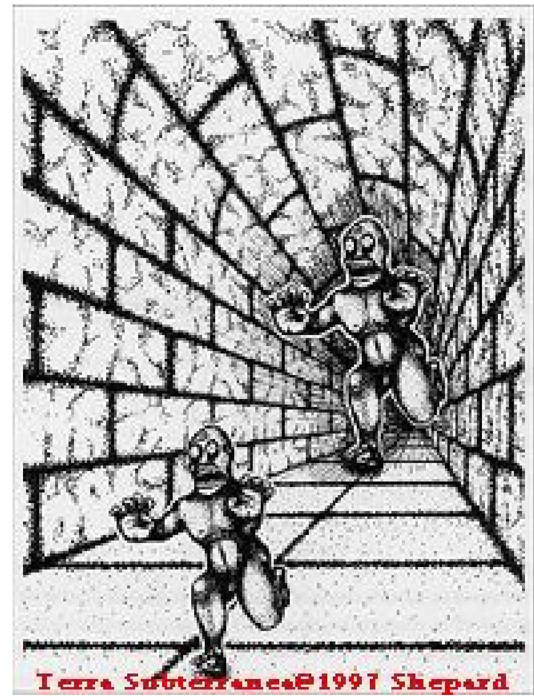
Necker cube





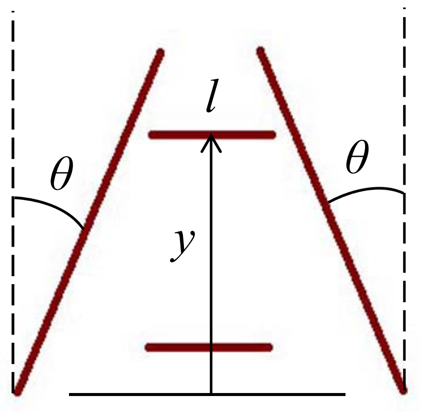
Ponzo illusion





Ponzo illusion

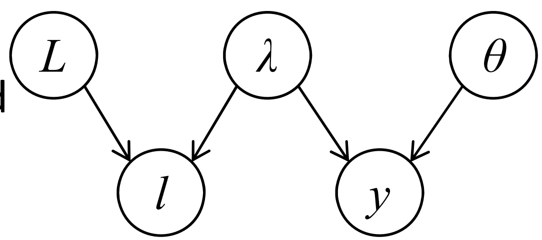
True length: *L*



$$
\begin{aligned}
p(L \mid l,\theta,y) &\propto p(l \mid L,\theta,y)\,p(L \mid \theta,y) \\
&= p(l \mid L,\theta,y)\,p(L) \\
&= p(L)\int p(l \mid L,\theta,y,\lambda)\,p(\lambda \mid \theta,y)\,d\lambda \\
&= p(L)\int p(l \mid L,\lambda)\,p(\lambda \mid \theta,y)\,d\lambda
\end{aligned}
$$

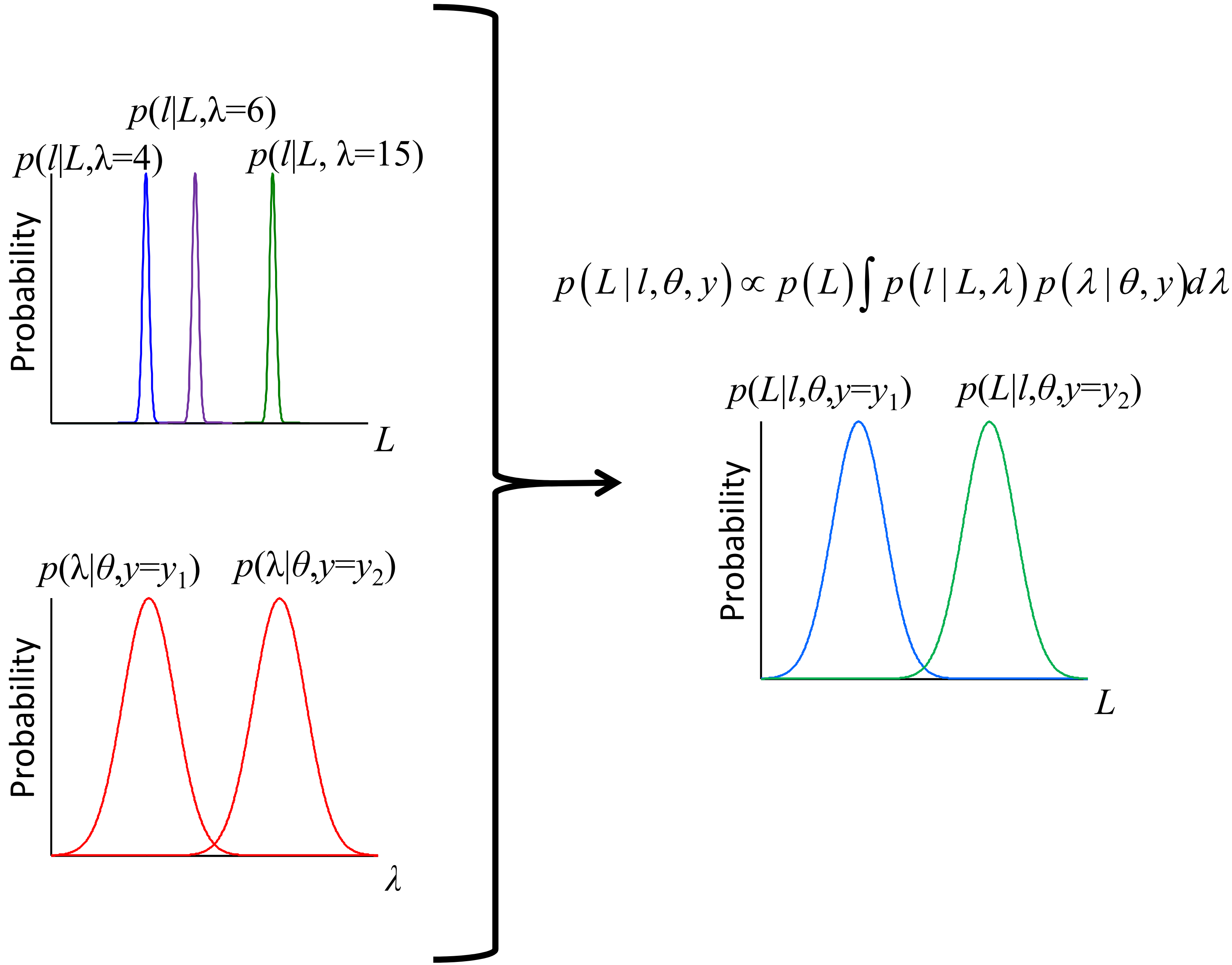
Write down a generative model for this task.

to be inferred



context

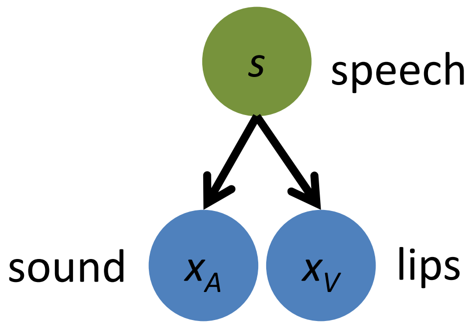
observations



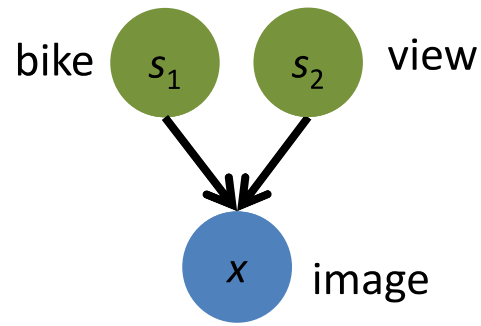
$p(l|L,\lambda=6)$
$p(l|L,\lambda=4)$
$p(l|L, \lambda=15)$
Probability
L
$p(\lambda|\theta,y=y_1)$
$p(\lambda|\theta,y=y_2)$
Probability
λ
$p\left(L \mid l,\theta,y\right) \propto p\left(L\right)\int p\left(l \mid L,\lambda\right) p\left(\lambda \mid \theta,y\right) d\lambda$
$p(L|l,\theta,y=y_1)$
$p(L|l,\theta,y=y_2)$
Probability
L

Elementary probability theory

- Joint probability $p(A,B)$
- Independence $p(A,B) = p(A)p(B)$ $\qquad \int p(A,B)\,dB = p(A)$
- Conditional probability $p(A \mid B) = \dfrac{p(A,B)}{p(B)}$
- Conditional independence $p(A,B|C) = p(A|C)\, p(B|C)$
- Marginalization $$p(A) = \int p(A \mid B)\, p(B)\, dB$$ $$p(A \mid C) = \int p(A \mid B, C)\, p(B \mid C)\, dB$$
- Bayes' rule $p(B \mid A) = \dfrac{p(A \mid B)\, p(B)}{p(A)} = \dfrac{p(A \mid B)\, p(B)}{\int p(A \mid B)\, p(B) dB}$



Cue integration

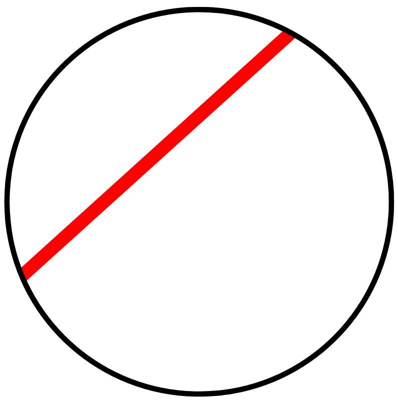


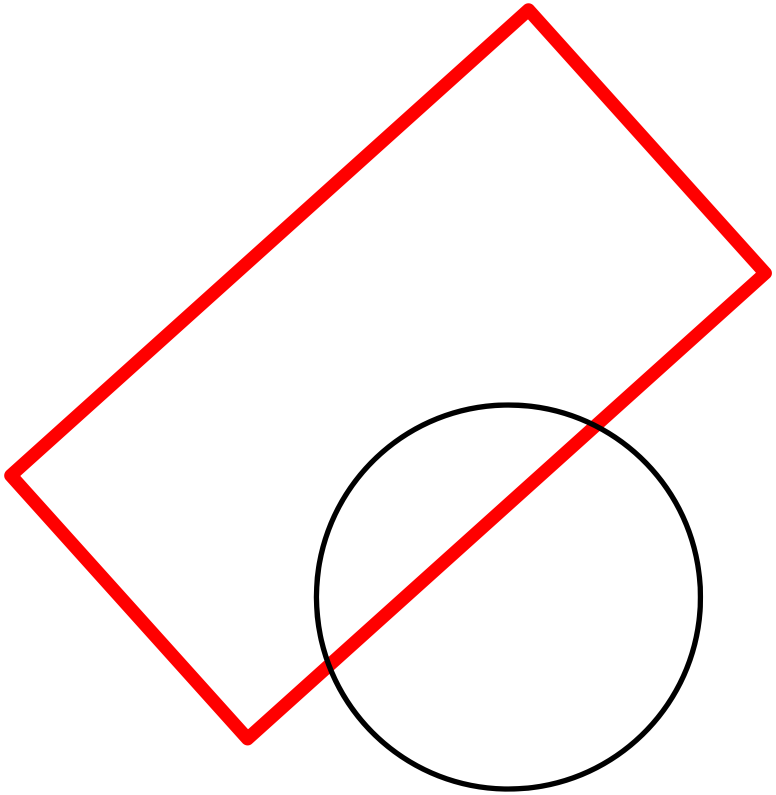
Invariant perception (discounting)

$$
\begin{aligned}
p(s \mid x_A, x_V) &\propto p(x_A, x_V \mid s)\, p(s) \\
&= p(x_A \mid s)\, p(x_V \mid s)\, p(s)
\end{aligned}
$$

$$
\begin{aligned}
&p(s_1 \mid x) \propto p(x \mid s_1)\, p(s_1) \\
&= p(s_1) \int p(x \mid s_1, s_2)\, p(s_2 \mid s_1)\, ds_2 \\
&= p(s_1) \int p(x \mid s_1, s_2)\, p(s_2)\, ds_2
\end{aligned}
$$

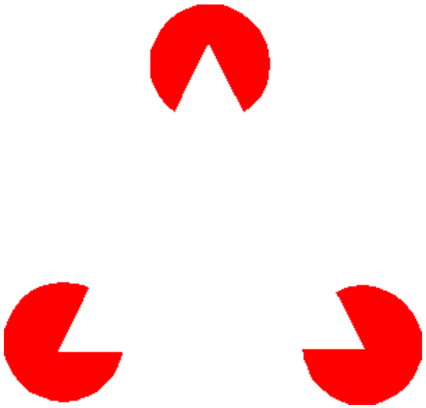
Aperture problem



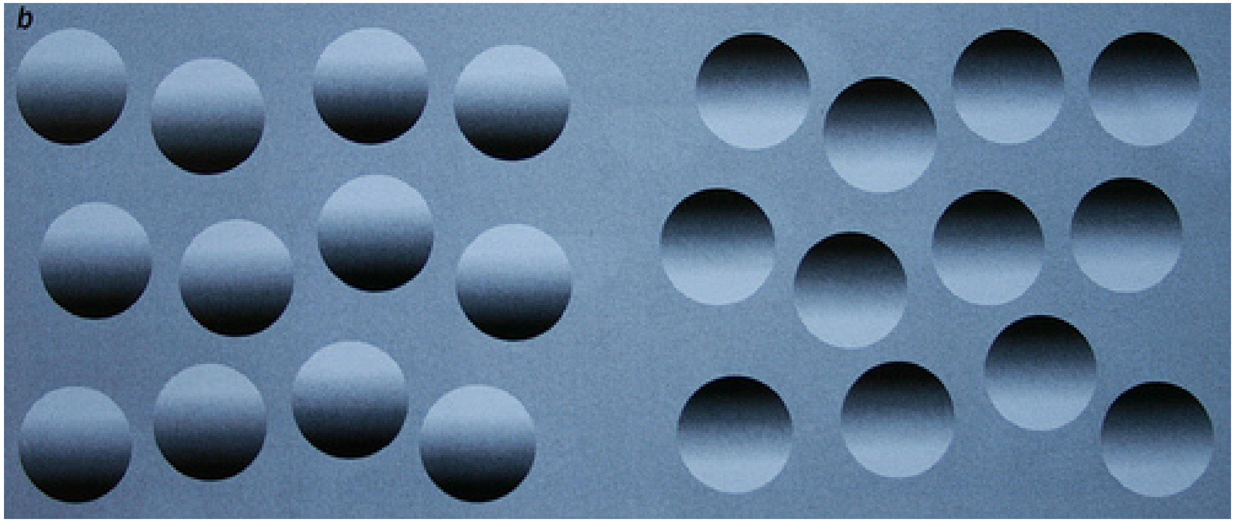


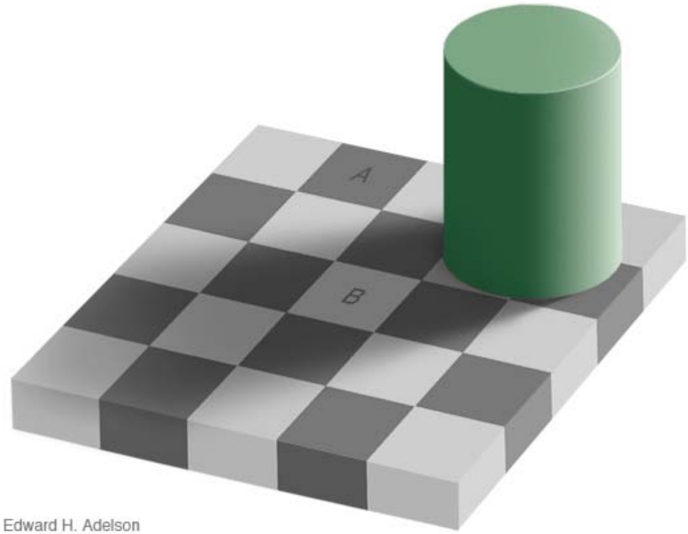
Exercise: Explain dominant percept in terms of a prior on low speeds.

Kanisza triangle

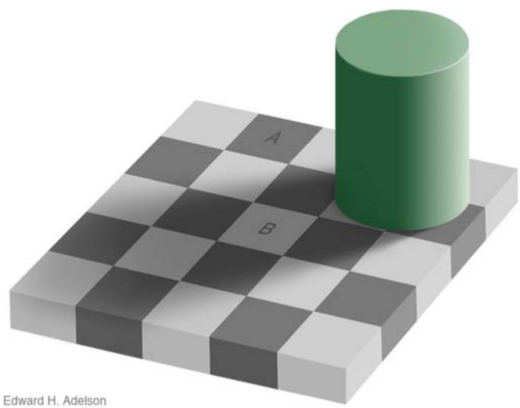


Convex or concave?

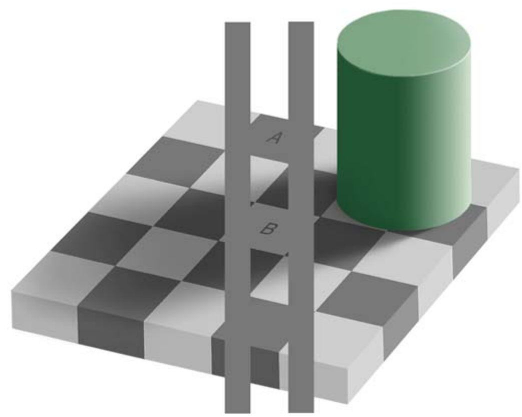




Edward H. Adelson



Edward H. Adelson



Misheard lyric:
“There’s a bathroom on the right.”

True: “There’s a bad moon on the rise.”

Marr and Bayes

- Computational: the problems vision must overcome
- Algorithmic: the strategy that is used
- Implementational: how it is done through neural operations

Summary

- Levels of analysis
- Sensory information is noisy and ambiguous.
- Variables like object identity have to be inferred based on sensory information and prior knowledge.
- Often, this inference involves manipulating multiple variables while keeping track of all probability distributions.
- Many illusions are byproducts of Bayesian inference.