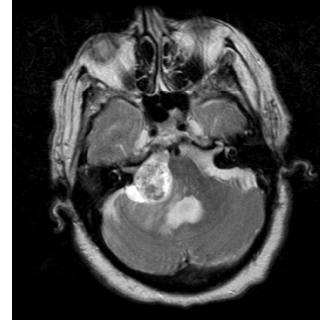
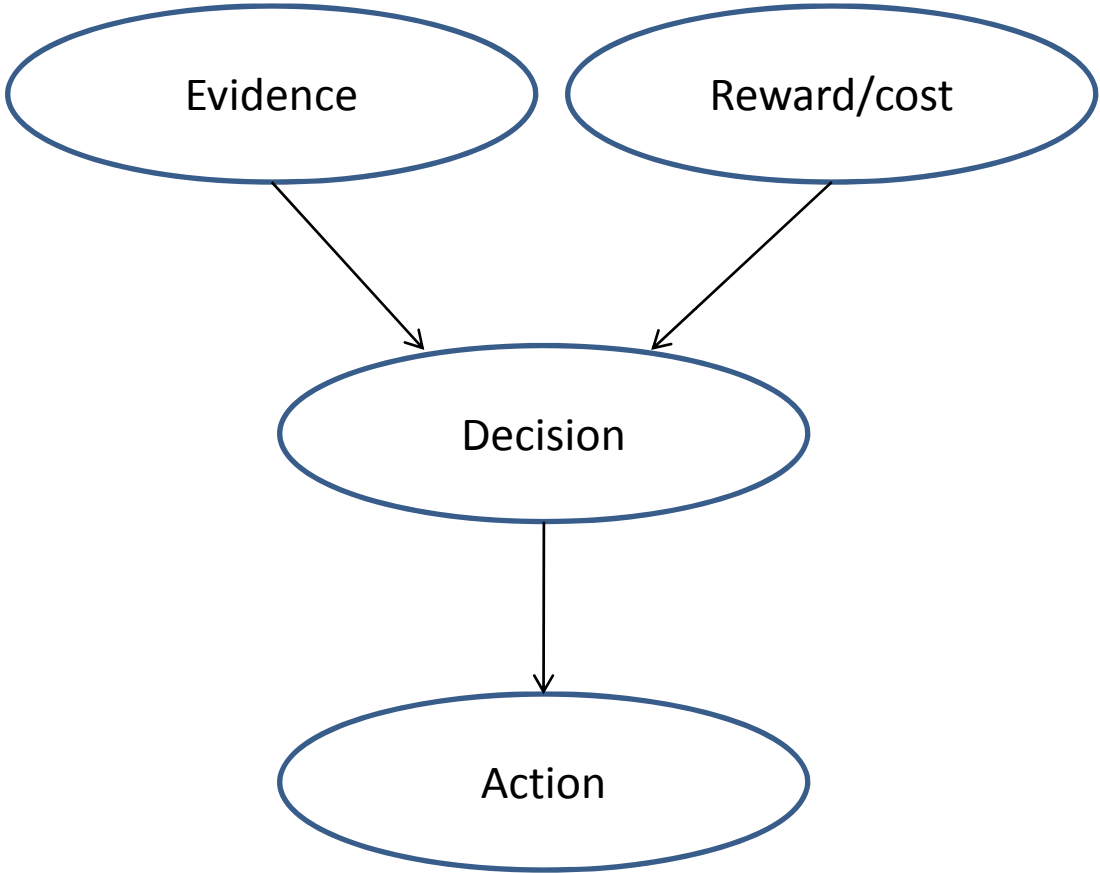


Decision-making

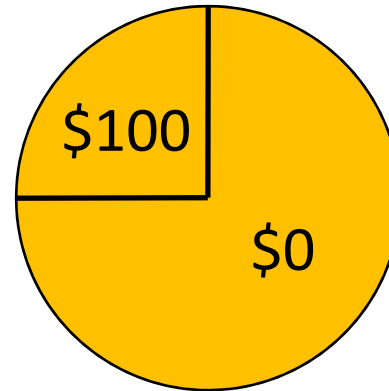
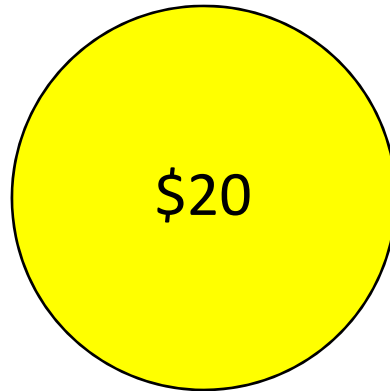
Lecture 7

Types of decisions

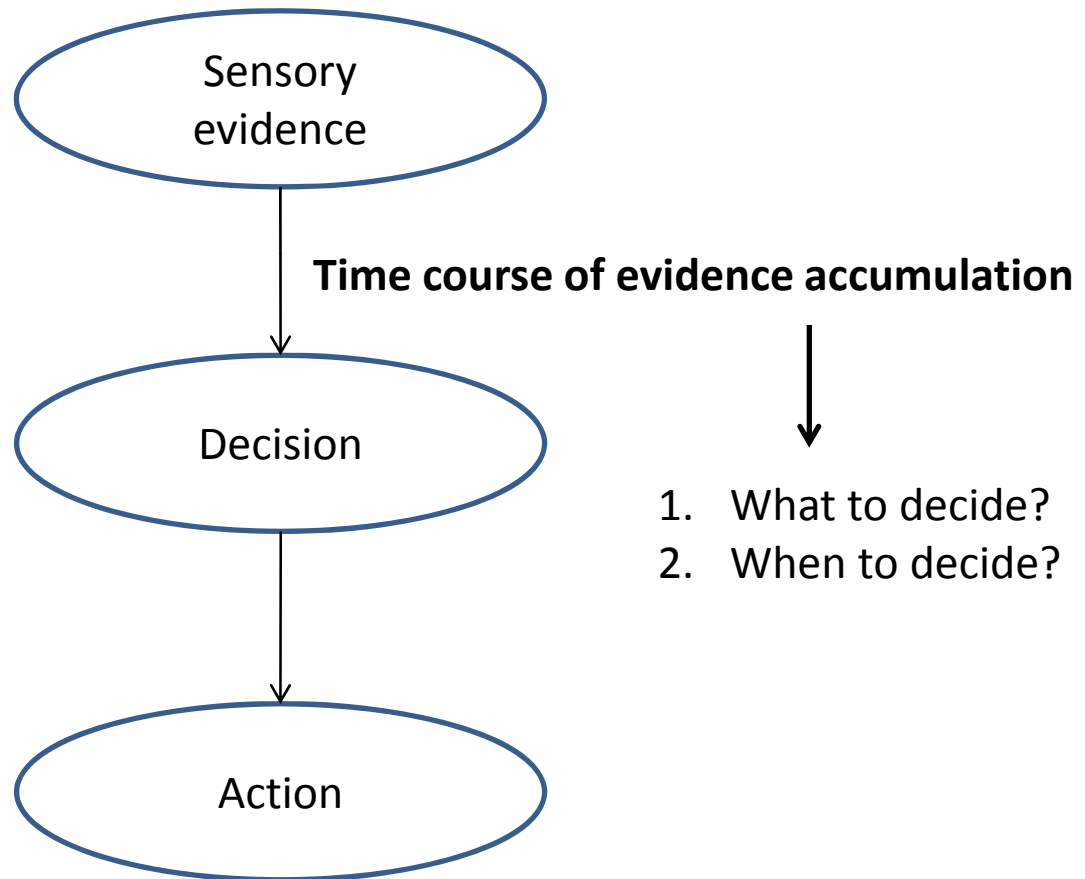




Reward-based decisions



Perceptual decisions



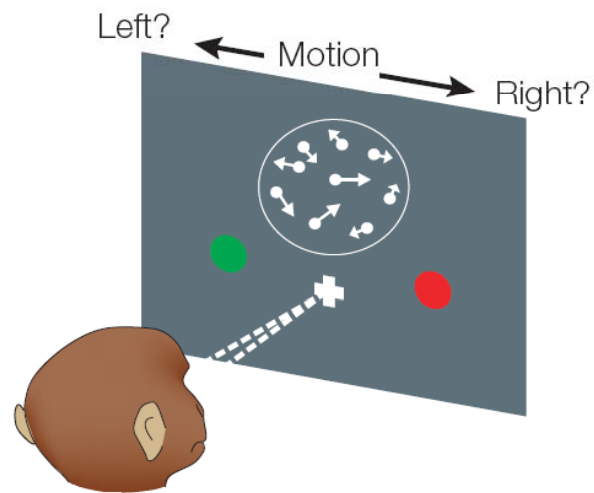
Multiple alternatives

APPETIZERS		
春卷	1.	Roast Pork Egg Roll (1)..... 1.00
春卷	2.	Shrimp Egg Roll (1)..... 1.10
豉汁蒸排骨	3.	Boneless Bar-B-Q Spare Ribs(Sm)4.85 (Lg)7.75
豉汁蒸排骨	4.	Bar-B-Q Spare Ribs(Sm)4.80 (Lg)9.20
豉汁蒸排骨	5.	Fantail Shrimp (1)..... 1.00
炸春卷	6.	Fried Wonton (10)..... 2.35
炸春卷	7.	Shrimp Toast (4)..... 2.65
炸春卷	8.	Fried Dumplings (7)..... (Order) 3.85
炸春卷	9.	Steamed Dumpling (7)..... (Order) 3.85
芝麻冷麵	9a.	Cold Sesame Noodles..... 4.50
牛肉	10.	Teriyaki Beef on Stick..... (3)3.35 (6)6.35
豉汁蒸排骨	10a.	Bar-B-Q Chicken on Stick..... (4)2.75 (8)5.25
豉汁蒸排骨	11.	* Fried Chicken Wings..... 3.75
豉汁蒸排骨	12.	Szechuan Sauce (12)..... 8.50
豉汁蒸排骨	12a.	Fried Shrimp..... 4.25
SOUPS (w. Fried Noodles)		
雲吞湯	10.	Wonton Soup..... 1.20 2.25
雲吞湯	14.	Egg Drop Soup..... 1.10 2.15
雲吞湯	15.	Wonton w. Egg Drop Soup..... 1.60 2.65
雲吞湯	16.	Chicken Noodle Soup..... 1.10 2.10
雲吞湯	17.	Chicken Rice Soup..... 1.10 2.10
雲吞湯	18.	House Special Soup..... 4.75
雲吞湯	19.	* Hot & Spicy Soup..... 1.75 3.35
雲吞湯	20.	Vegetable Soup..... 1.10 2.10
雲吞湯	21.	Shrimp or Beef Yat Gaw Mein..... 3.75
雲吞湯	22.	Chicken or Pork Yat Gaw Mein..... 3.50
CHOW MEIN		
(w. Fried Noodles and White Rice)		
豉汁炒麵	23.	Chicken Chow Mein..... 2.80 5.40
豉汁炒麵	24.	Pork Chow Mein..... 2.80 5.40
豉汁炒麵	25.	Beef Chow Mein..... 3.00 5.60
豉汁炒麵	26.	Shrimp Chow Mein..... 3.35 6.60
豉汁炒麵	27.	Vegetable Chow Mein..... 2.30 4.20
豉汁炒麵	28.	Subgum Chicken Chow Mein..... 3.25 6.20
豉汁炒麵	29.	Subgum Shrimp Chow Mein..... 3.50 6.60
豉汁炒麵	30.	Lobster Chow Mein..... 4.75 9.20
豉汁炒麵	31.	House Special Chow Mein..... 3.75 7.20
CHOP SUEY		
(w. White Rice)		
豉汁炒麵	32.	Pork Chop Suey..... 2.85 5.50
豉汁炒麵	33.	Chicken Chop Suey..... 2.95 5.80
豉汁炒麵	34.	Vegetable Chop Suey..... 2.50 4.80
豉汁炒麵	35.	Beef Chop Suey..... 3.35 6.35
豉汁炒麵	36.	Shrimp Chop Suey..... 3.65 6.95
豉汁炒麵	37.	Lobster Chop Suey..... 4.85 9.30
豉汁炒麵	38.	House Special Chop Suey..... 3.75 7.20
FRIED RICE		
(w. White Rice)		
豉汁炒麵	39.	Roast Pork Fried Rice..... 2.80 5.30
豉汁炒麵	40.	Chicken Fried Rice..... 2.90 5.45
豉汁炒麵	41.	Vegetable Fried Rice..... 2.45 4.60
豉汁炒麵	42.	Beef Fried Rice..... 3.25 6.15
豉汁炒麵	43.	Shrimp Fried Rice..... 3.45 6.75
豉汁炒麵	44.	Lobster Fried Rice..... 4.70 8.85
豉汁炒麵	45.	Young Chow Fried Rice..... 3.55 6.80
豉汁炒麵	45a.	Plain Fried Rice..... 2.15 4.15
LO MEIN (Soft Noodles)		
豉汁炒麵	46.	Vegetable Lo Mein..... 2.50 4.65
豉汁炒麵	47.	Roast Pork Lo Mein..... 3.10 6.00
豉汁炒麵	48.	Chicken Lo Mein..... 3.20 6.15
豉汁炒麵	49.	Beef Lo Mein..... 3.35 6.25
豉汁炒麵	50.	Shrimp Lo Mein..... 3.65 6.95
豉汁炒麵	51.	Lobster Lo Mein..... 4.75 8.55
豉汁炒麵	52.	House Special Lo Mein..... 4.25 7.95
PORK		
(w. White Rice)		
豉汁炒麵	53.	Roast Pork w. Chinese Veg..... 3.20 6.20
豉汁炒麵	54.	Roast Pork w. Pepper Onion..... 3.50 6.80
豉汁炒麵	55.	Roast Pork w. Snow Peas..... 3.90 7.50
豉汁炒麵	56.	Roast Pork w. Broccoli..... 3.75 7.20
豉汁炒麵	57.	Roast Pork w. Almond Ding..... 3.75 7.20
豉汁炒麵	58.	Roast Pork w. Mushroom..... 3.75 7.20
豉汁炒麵	59.	Roast Pork w. Oyster Sauce..... 3.79 7.20
豉汁炒麵	59a.	Roast Pork w. Mixed Vegetable..... 3.85 7.35
CHICKEN		
(w. White Rice)		
豉汁炒麵	60.	Chicken w. Chinese Veg..... 3.95 6.90
豉汁炒麵	61.	Moo Goo Gai Pan..... 3.45 6.55
豉汁炒麵	62.	Chicken w. Snow Peas..... 3.95 7.95
豉汁炒麵	63.	Chicken w. Broccoli..... 3.85 7.35
豉汁炒麵	64.	Chicken w. Almond Ding..... 3.85 7.25
豉汁炒麵	65.	Chicken w. Black Bean Sc..... 3.75 7.20
豉汁炒麵	66.	Chicken w. Oyster Sauce..... 3.75 7.20
豉汁炒麵	67.	* Chicken w. Curry Sauce..... 3.95 7.35
豉汁炒麵	67a.	Plain Broccoli..... 3.25 6.00
豉汁炒麵	67b.	Chicken w. Mixed Vegetable..... 4.00 7.75
BEEF		
(w. White Rice)		
豉汁炒麵	68.	Beef w. Chinese Veg..... 3.85 6.95
豉汁炒麵	69.	Pepper Steak w. Onion..... 3.95 7.50
豉汁炒麵	70.	Beef w. Pepper & Tomato..... 3.95 7.25
豉汁炒麵	71.	Beef w. Snow Peas..... 4.20 7.85
豉汁炒麵	72.	Beef w. Broccoli..... 4.20 7.85
豉汁炒麵	73.	Beef w. Mushroom..... 4.00 7.70
豉汁炒麵	74.	Beef w. Oyster Sauce..... 4.00 7.70
豉汁炒麵	75.	* Curry Beef w. Onion..... 4.00 7.70
豉汁炒麵	75a.	Beef w. Mixed Vegetables..... 4.00 7.75
SHRIMP		
(w. White Rice)		
豉汁炒麵	76.	Lobster Sauce..... 2.25 4.20
豉汁炒麵	77.	Shrimp w. Lobster Sauce..... 4.55 8.70
豉汁炒麵	78.	Shrimp w. Chinese Veg..... 4.45 8.50
豉汁炒麵	79.	Shrimp w. Pepper & Tomato..... 4.55 8.70
豉汁炒麵	80.	Shrimp w. Broccoli..... 4.55 8.70
豉汁炒麵	81.	Shrimp w. Almond Ding..... 4.25 8.20
豉汁炒麵	82.	Shrimp w. Snow Peas..... 4.75 9.20
豉汁炒麵	83.	Shrimp w. Mushrooms..... 4.50 8.70
豉汁炒麵	84.	* Shrimp w. Curry Sauce..... 4.50 8.70
豉汁炒麵	85.	Shrimp w. Black Bean Sauce..... 4.50 8.70
豉汁炒麵	85a.	Shrimp w. Mixed Vegetables..... 4.65 8.75
EGG FOO YOUNG		
(w. White Rice)		
豉汁炒麵	86.	Muchroom Egg Foo Young..... 4.95
豉汁炒麵	87.	Roast Pork Egg Foo Young..... 5.00
豉汁炒麵	88.	Chicken Egg Foo Young..... 5.00
豉汁炒麵	89.	Shrimp Egg Foo Young..... 6.45
豉汁炒麵	90.	House Special Egg Foo Young..... 6.25
SWEET & SOUR		
(w. White Rice)		
豉汁炒麵	91.	Sweet & Sour Pork..... 3.90 7.35
豉汁炒麵	92.	Sweet & Sour Chicken..... 3.90 7.35
豉汁炒麵	93.	Sweet & Sour Shrimp..... 4.50 8.70
豉汁炒麵	94.	Sweet & Sour Combination (Order) 7.95
豉汁炒麵	95.	French Fries..... (Sm.) 1.35 (Lg.) 2.55
豉汁炒麵	96.	Fortune Cookies..... (Per Bag) 0.50
豉汁炒麵	97.	Almond Cookies..... (Per Bag) 0.50
豉汁炒麵	98.	Finest Home Made Crispy Noodles..... 0.50
豉汁炒麵	99.	Boiled Rice..... (Ft.) 0.75 (Qt.) 1.50
* HOT & SPICY		

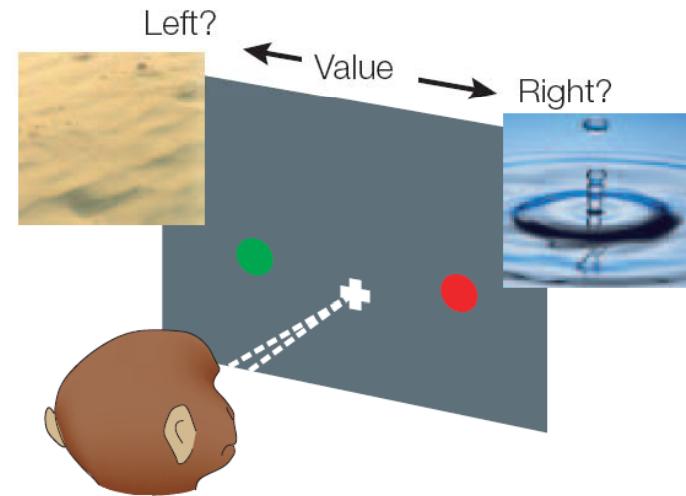


Physiology: binary choice

a Perceptual discrimination task



c Free-choice task

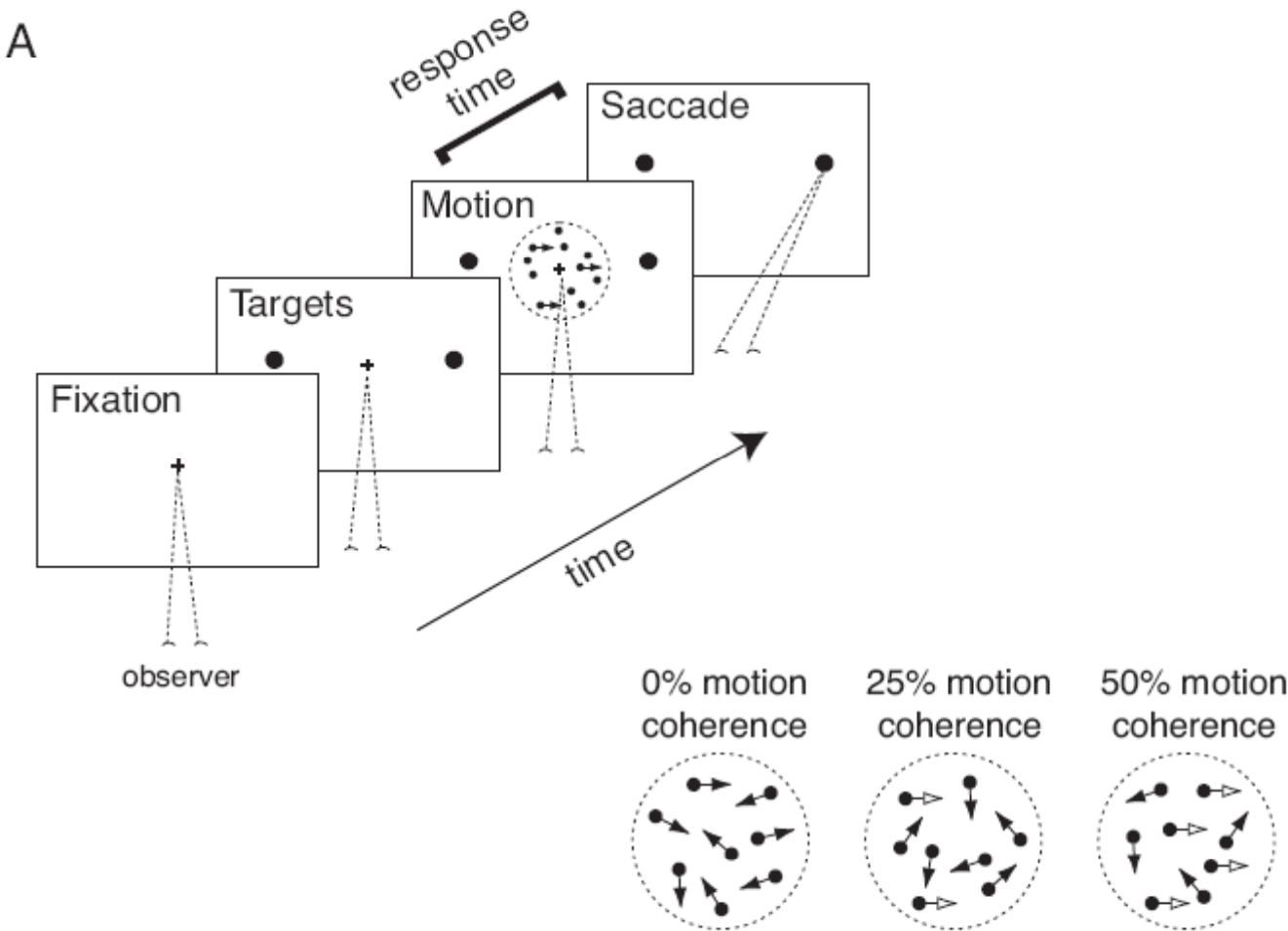


Sugrue, Corrado, Newsome, 2005

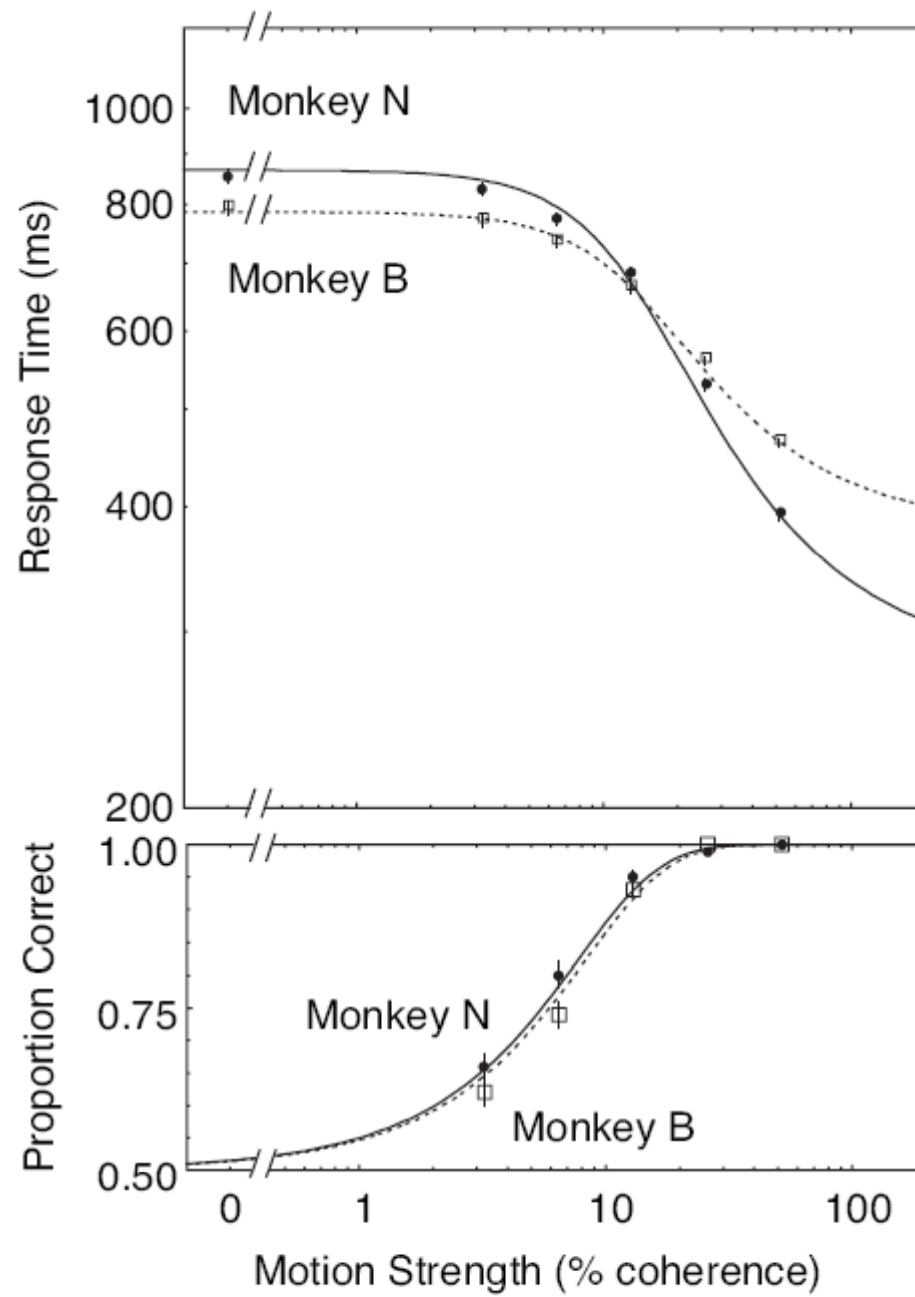
Types of experiments

- Response time procedure (free response protocol): unlimited time
- Response signal procedure (interrogation protocol): limited time

Motion discrimination – response time



Palmer, Huk, Shadlen 2005

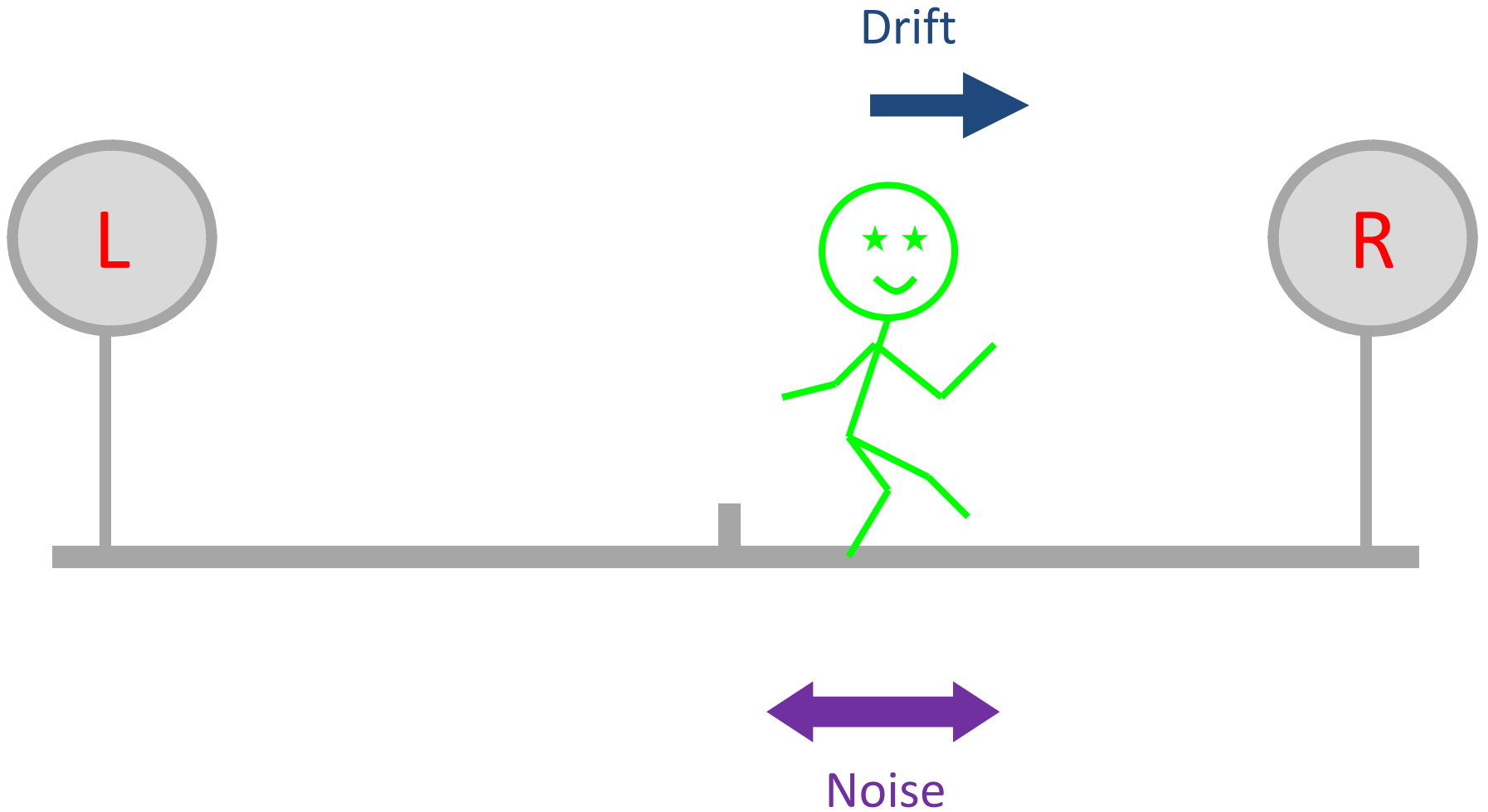


Types of models

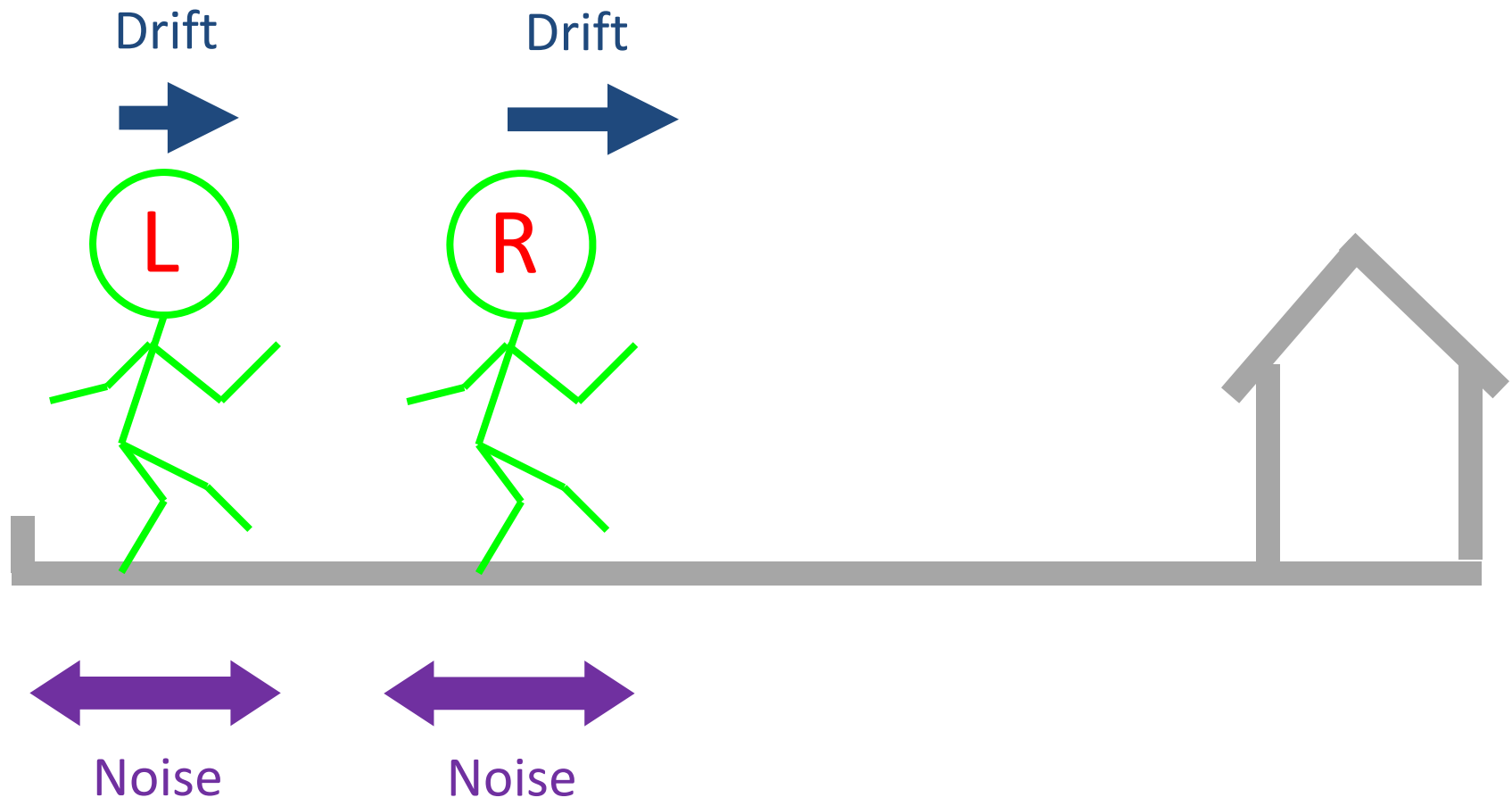
- Diffusion models
- Race (accumulator) models
- Bayesian models

- Behavioral models
- Neural models

Diffusion model



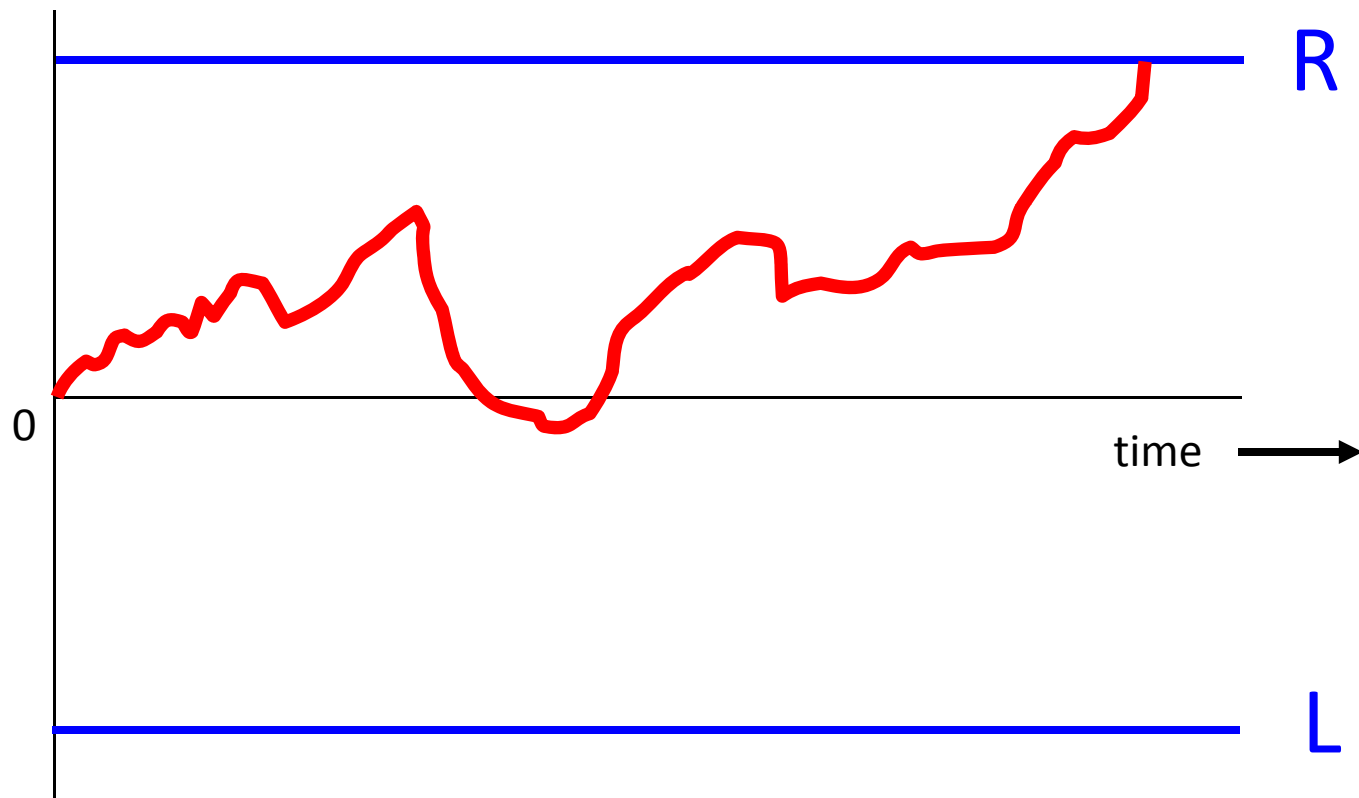
Race model

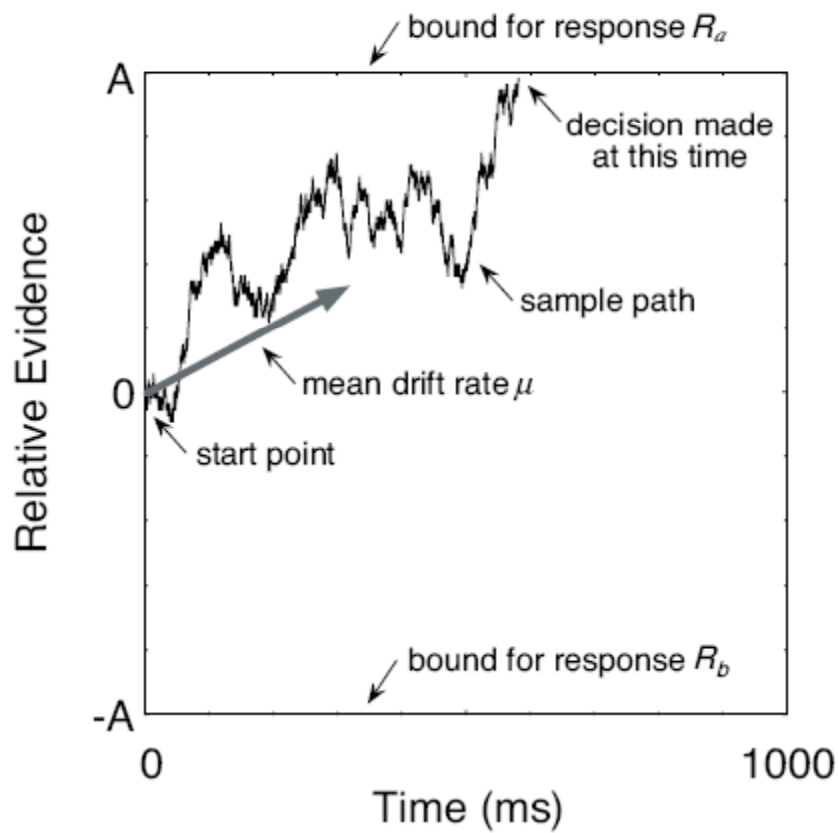


Pros and cons

- **Diffusion models** (Wald; Gold/Shadlen; Ratcliff)
 - Simple
 - Good description of psychophysics
 - What is the decision variable?
- **Accumulator models** (Usher/McClelland; McMillen/Holmes)
 - Neural flavor
 - More parameters
 - Worse fit to the data?

Diffusion model



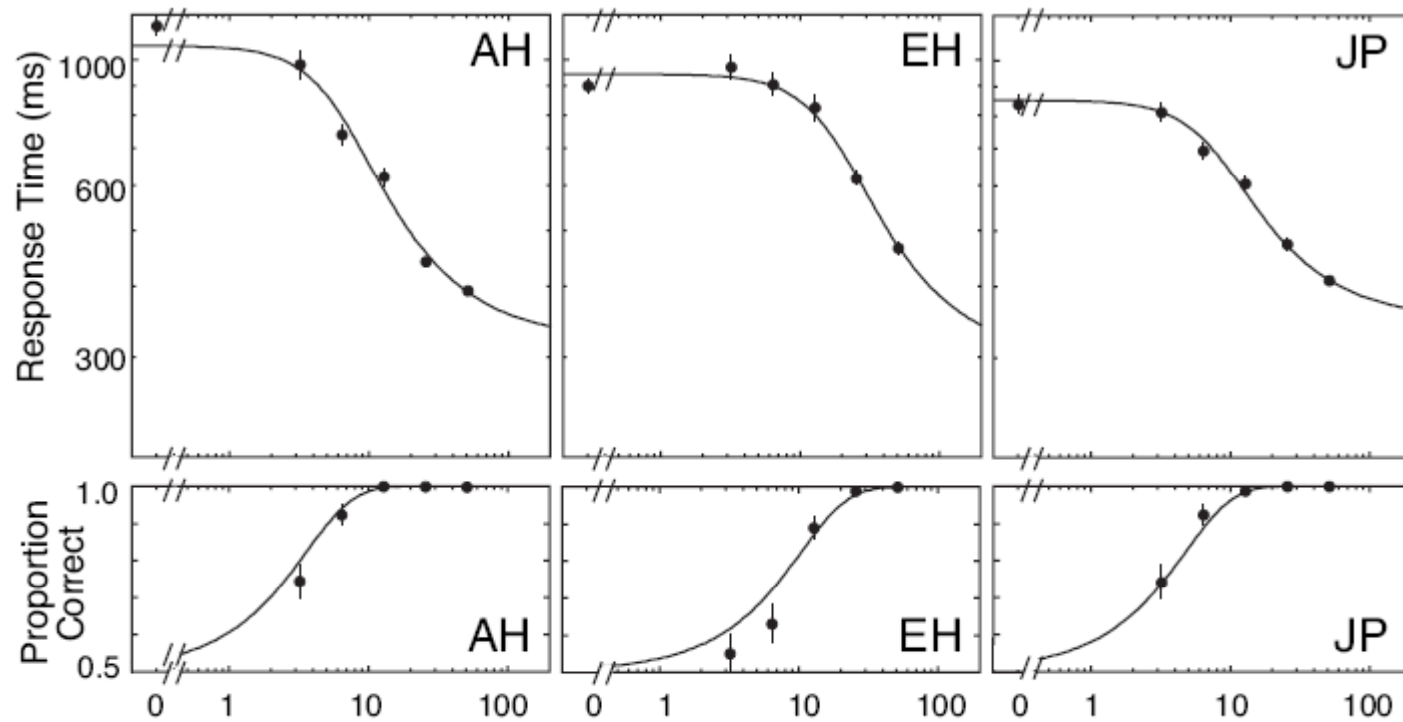


$$\mu' = kx$$

$$P_C(x) = \frac{1}{1 + e^{-2A'k|x|}}$$

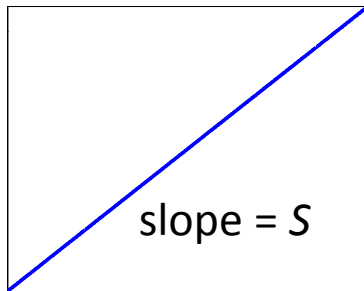
$$t_T(x) = \frac{A'}{kx} \tanh(A'kx) + t_R$$

Binary choice – response time



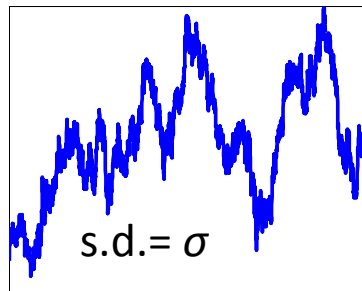
Leaky accumulator model

Integration



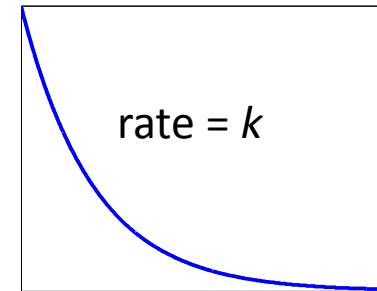
$$\frac{dx}{dt} = S$$

White noise



$$\frac{dx}{dt} = \text{white noise}$$

Leakage



$$\frac{dx}{dt} = -kx$$

Leaky accumulator model

$$dx_i = \left(\quad \quad \quad \right) dt$$



i 'th unit's change
in time dt

Leaky accumulator model

$$dx_i = \left(S_i \right) dt$$

i 'th signal
(drift)

Leaky accumulator model

$$dx_i = \left(S_i - kx_i \right) dt$$



i'th unit's leakage

Leaky accumulator model

$$dx_i = \left(S_i - kx_i \right) dt + \sigma dW_i$$



white noise
into i 'th unit

Leaky accumulator model

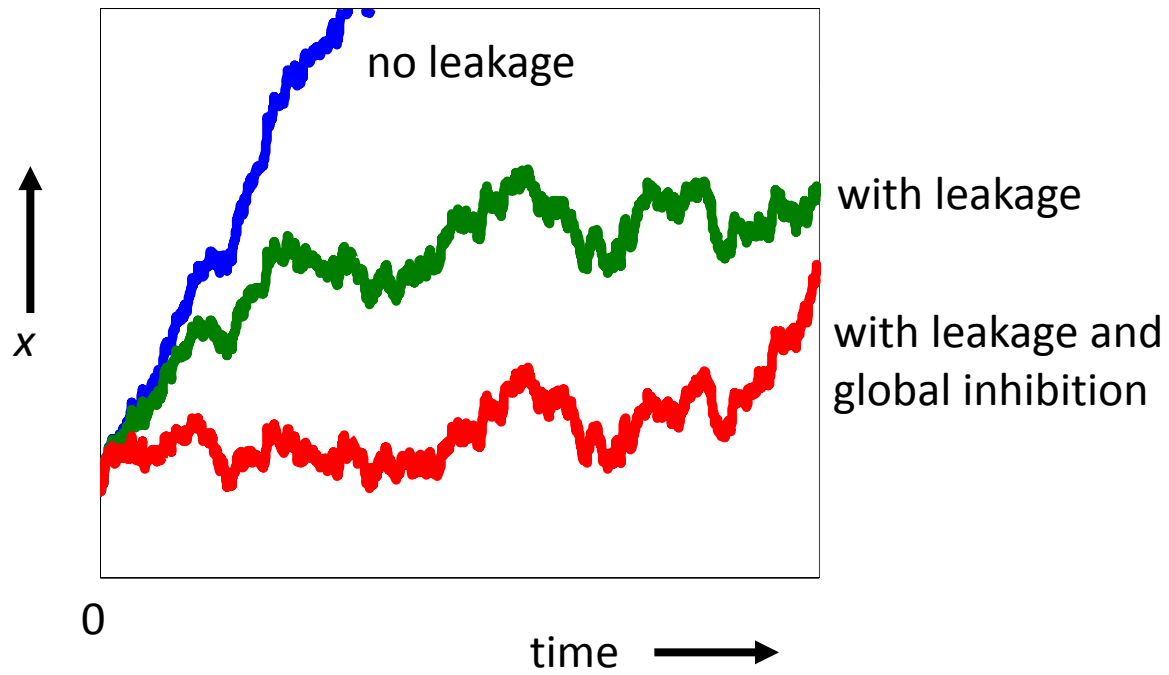
$$dx_i = \left(S_i - kx_i - w \sum_{j \neq i} x_j \right) dt + \sigma dW_i$$



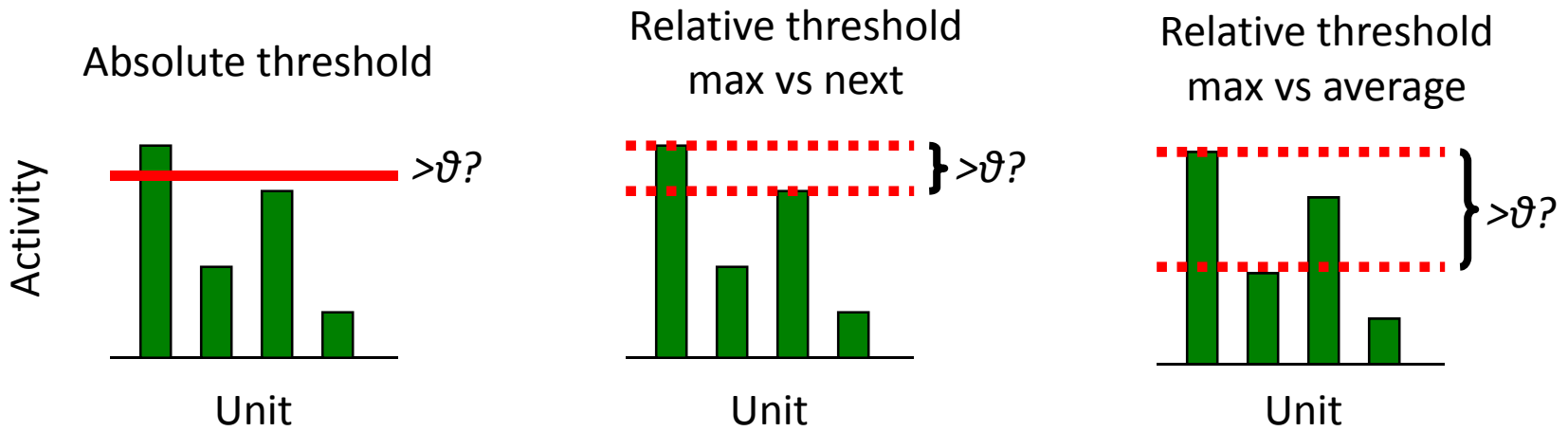
global inhibition

Leaky accumulator model

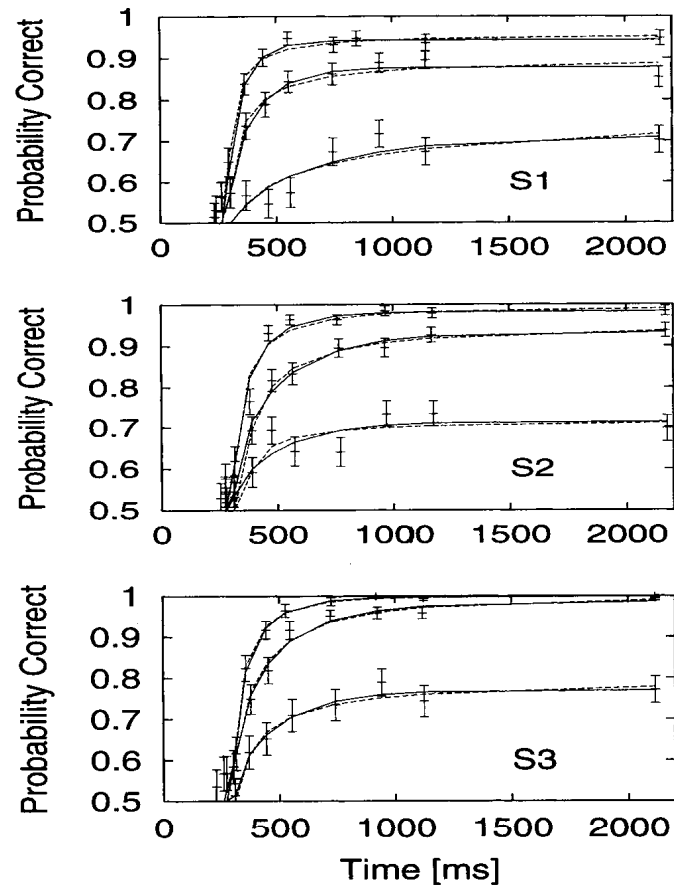
Typical behavior



When to decide?



Binary choice – Response signal



Problems with existing models

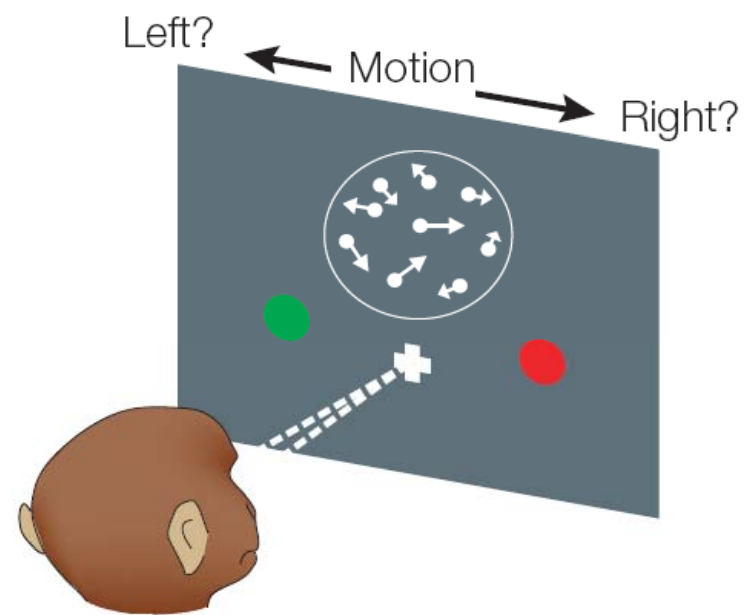
- Limited to binary or discrete choice
- Uncertainty not encoded
- What if certainty changes over time or over trials?
- No normative solution

Stages of decision-making

Two stages to decision-making:

1. Accumulating the evidence
2. Action selection

- Can we build a neural network that performs accumulation and action selection optimally for any number of choices? No neural model is known beyond two choices.
- Would this network account for existing data?



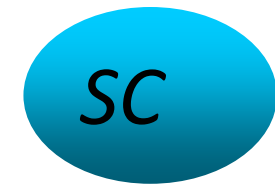
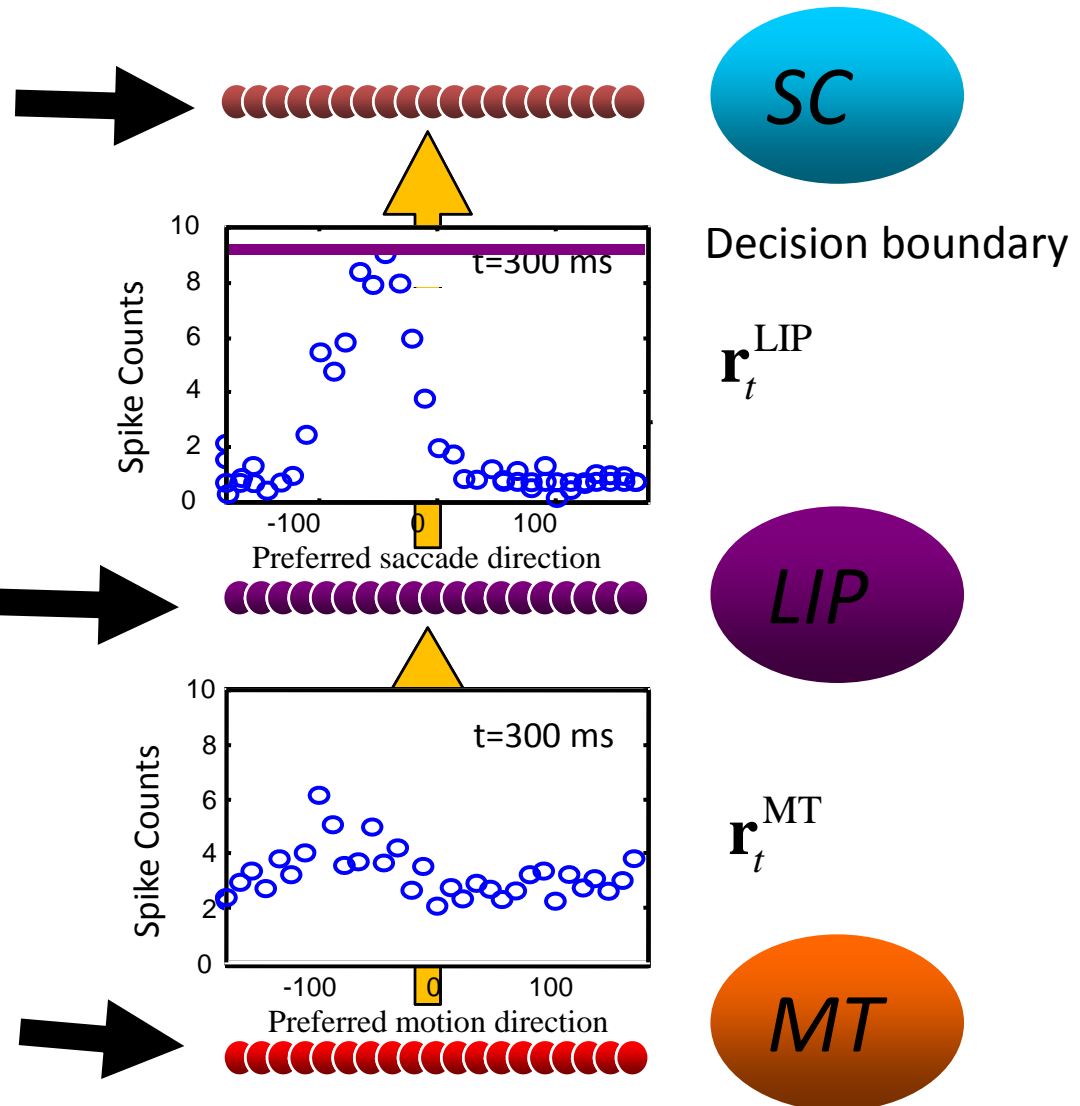
Single Trial Network Architecture Activity

Action selection : attractor dynamics

Integration: 100 spiking (LNP) neurons with lateral connections and long time constant

Termination rule: Stop integration when peak activity reaches a preset bound

Sensory data: 100 direction-selective neurons



Decision boundary

$$\mathbf{r}_t^{\text{LIP}}$$



$$\mathbf{r}_t^{\text{MT}}$$



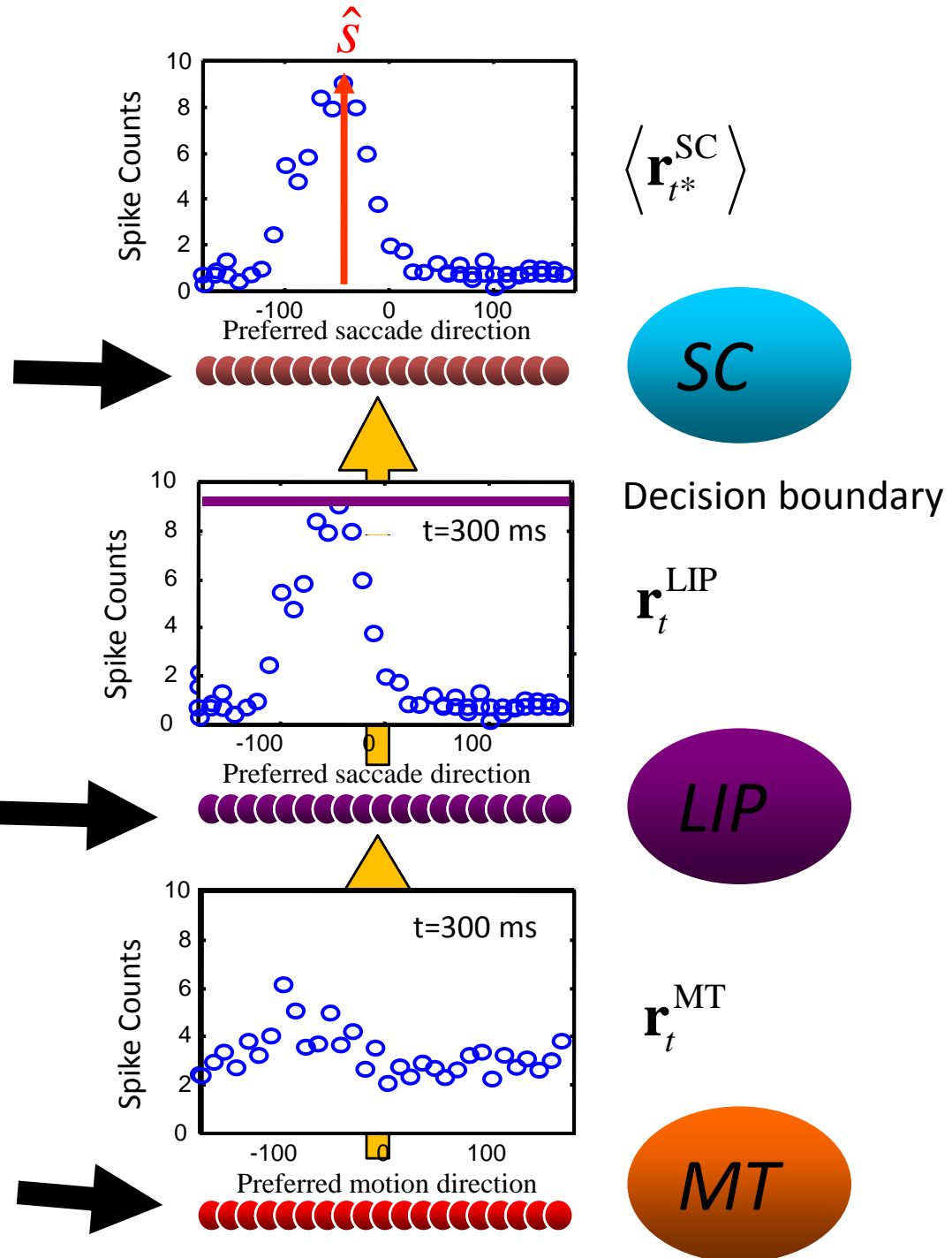
Single-Trial Activity

Action selection : attractor dynamics

Integration: 100 spiking (LNP) neurons with lateral connections and long time constant

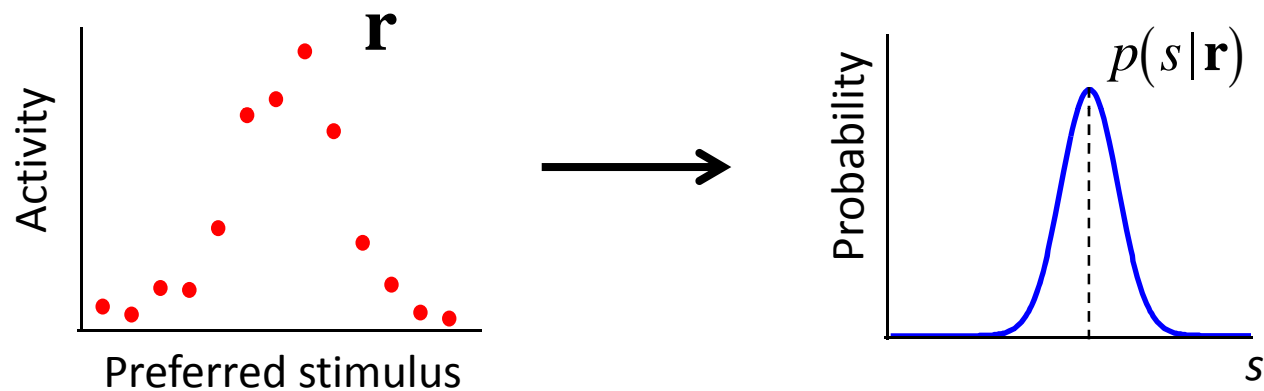
Termination rule: Stop integration when peak activity reaches a preset bound

Sensory data: 100 direction-selective neurons



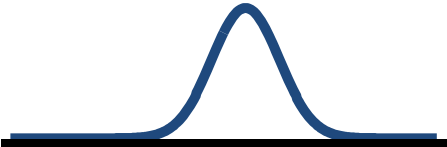
Why is such a network near-optimal?

Probability distributions from neural activity

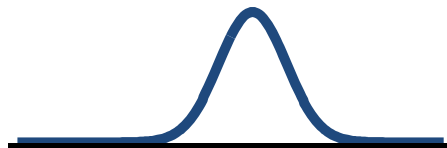


$$p(s|\mathbf{r}) \propto p(\mathbf{r}|s)p(s)$$

Time 1 evidence



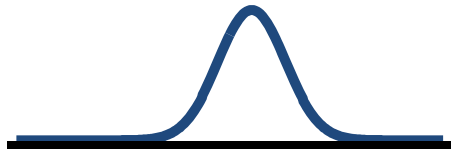
Time 2 evidence



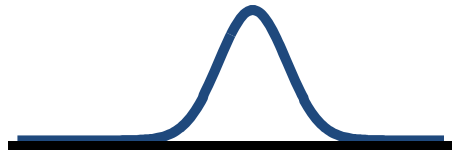
Time 3 evidence



Time 1 evidence $p(s | t = 1)$



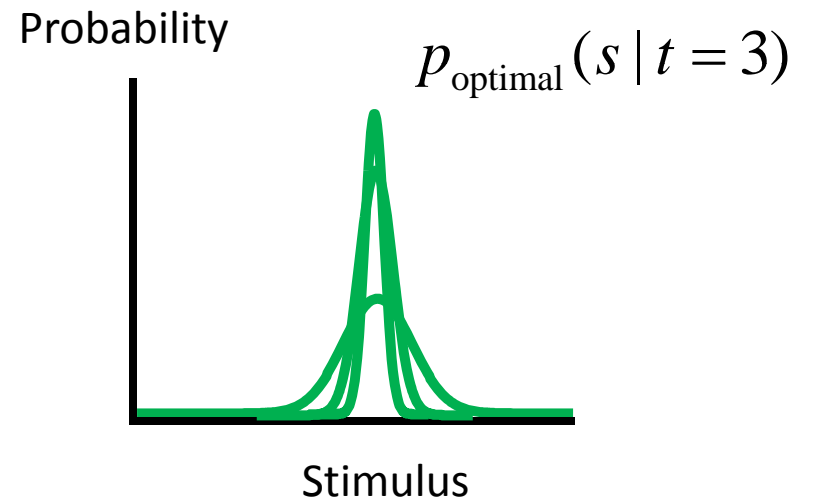
Time 2 evidence $p(s | t = 2)$



Time 3 evidence $p(s | t = 3)$



Optimal evidence
accumulation



Bayesian evidence accumulation

Given the pattern of activity from MT since the beginning of a trial, the optimal decision should be based on the posterior distribution:

$$p(s | \mathbf{r}_t^{\text{MT}}, \dots, \mathbf{r}_1^{\text{MT}}) \propto p(\mathbf{r}_t^{\text{MT}} | s) \dots p(\mathbf{r}_2^{\text{MT}} | s) p(\mathbf{r}_1^{\text{MT}} | s)$$

How can LIP compute and represent this probability distribution?

Poisson-like neural variability

$$p(\mathbf{r} | s) = \varphi(\mathbf{r}) e^{\mathbf{h}(s) \cdot \mathbf{r}}$$

- Includes independent Poisson
- Allows for Fano factors different from 1
- Allows for correlated variability
- Kernel $\mathbf{h}(s)$ determined by tuning curves and correlation structure of population
- Makes optimal cue integration easy to implement

Integration over time is optimal

in decision area

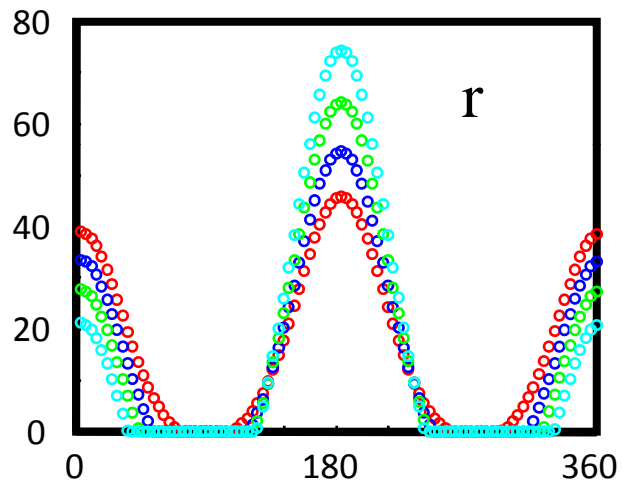
sensory input

$$p(s | \mathbf{r}_{t=1} + \dots + \mathbf{r}_{t=T}) \propto p(s | \mathbf{r}_{t=1}) \cdots p(s | \mathbf{r}_{t=T})$$

- Integrating neural activity over time corresponds to multiplying the encoded probability distributions, which is the condition for optimality.
- If LIP performs temporal integration, it encodes the optimal posterior distribution at all times.

Binary Choice: Distribution Encoded in LIP

Firing rate (Hz)

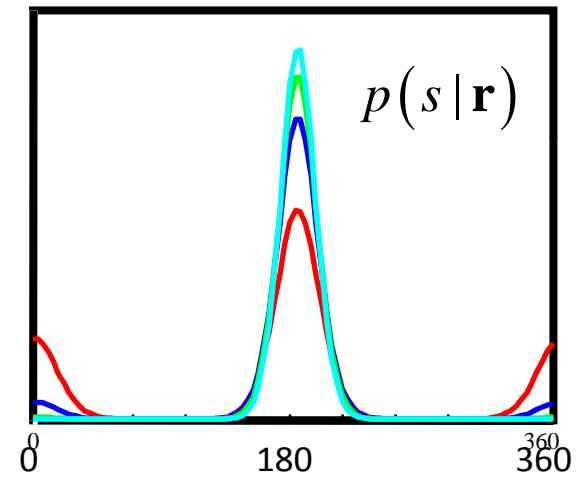


Preferred saccade direction (°)

OUT

IN

Probability



Saccade direction (°)

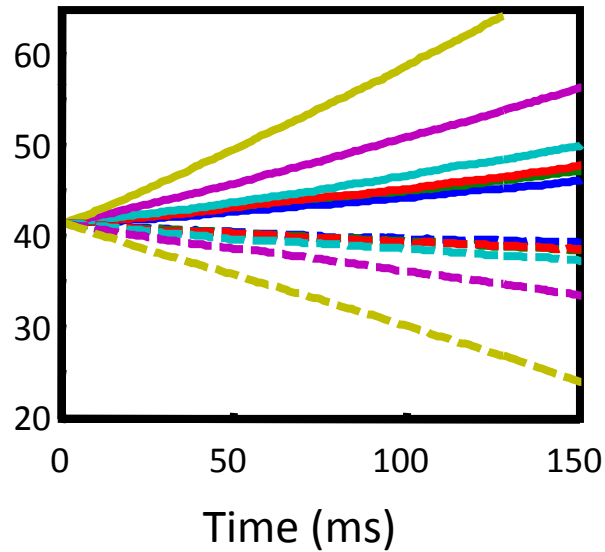
Bayes



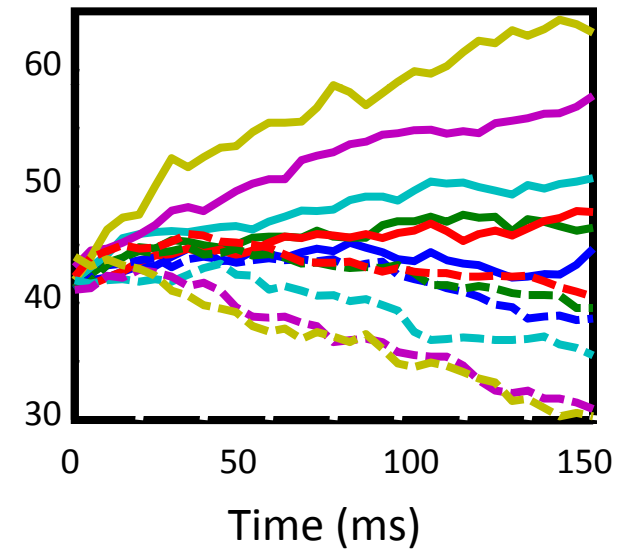
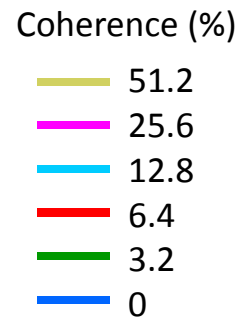
- 200 ms
- 150 ms
- 100 ms
- 50 ms

Binary Choice: Activity of IN and OUT neurons

Firing rate (Hz)



Firing rate (Hz)

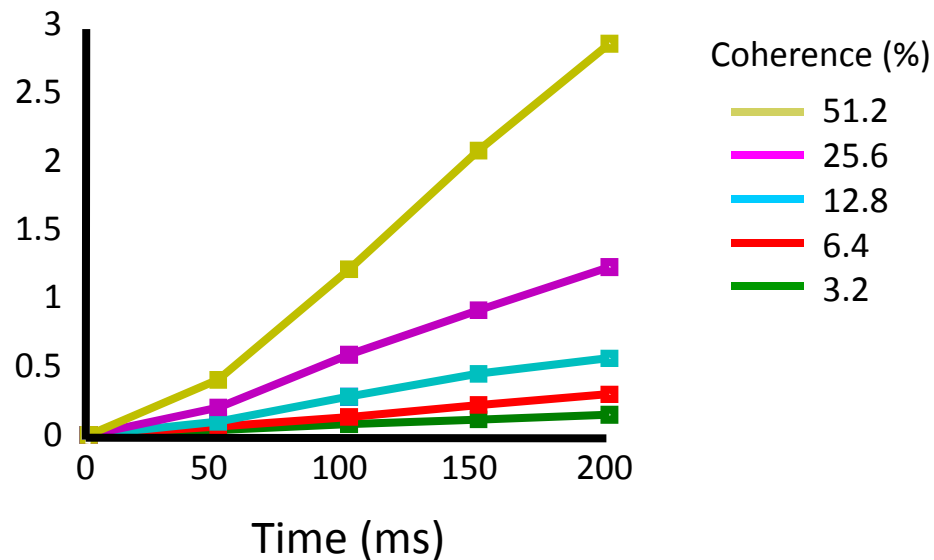


Data from Roitman and Shadlen (2002)

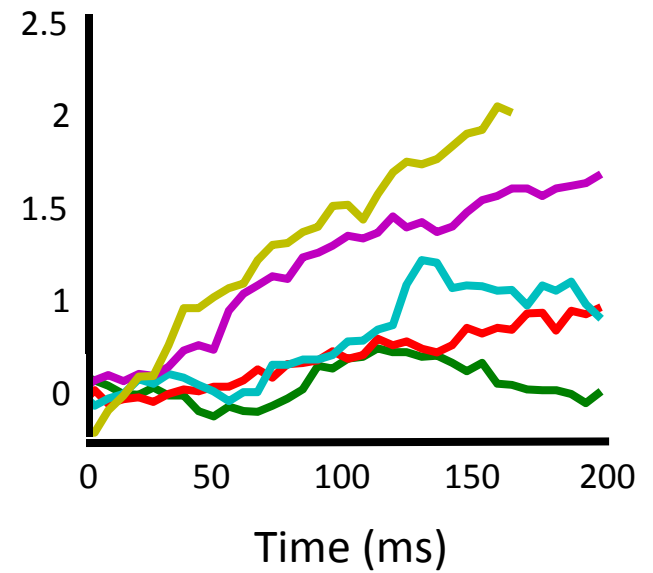
Log odds grow linearly in time

$$\log \frac{p(s = 0^\circ | r_0, r_{180})}{p(s = 180^\circ | r_0, r_{180})} \propto r_0 - r_{180}$$

Log odds



Log odds



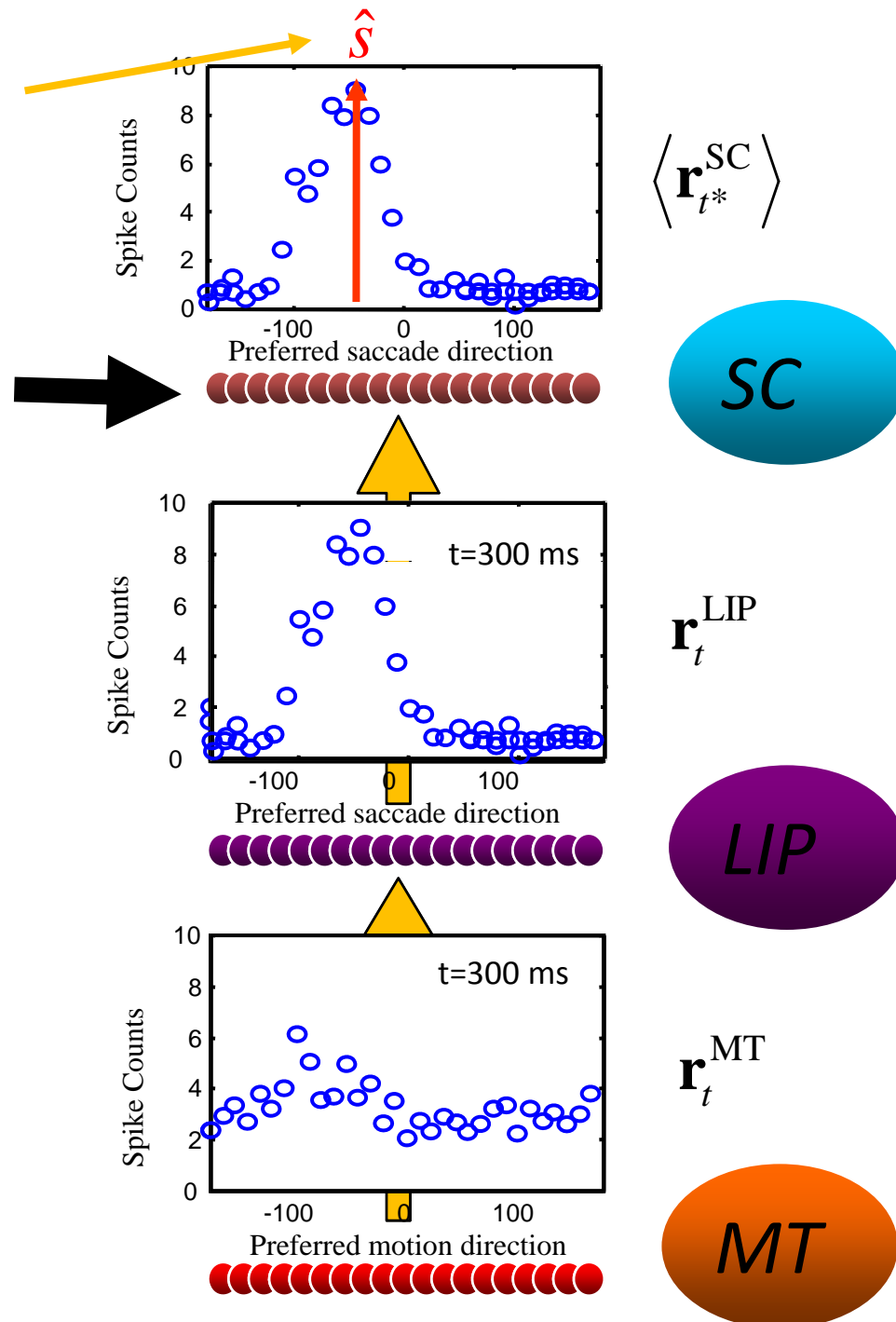
Optimal Action Selection

- LIP encodes the optimal posterior distribution at all times. But how is it read out optimally?
- Which action is the most likely to be right → maximum-likelihood estimate
- Can the SC take LIP activity at decision time and generate the maximum-likelihood saccade?
- For general noise distributions, the answer is no.
- However, if the noise is Poisson-like, then a solution exists.
- A network with a line attractor can be tuned to recover the maximum-likelihood estimate.

(Deneve, Latham, Pouget 1999)

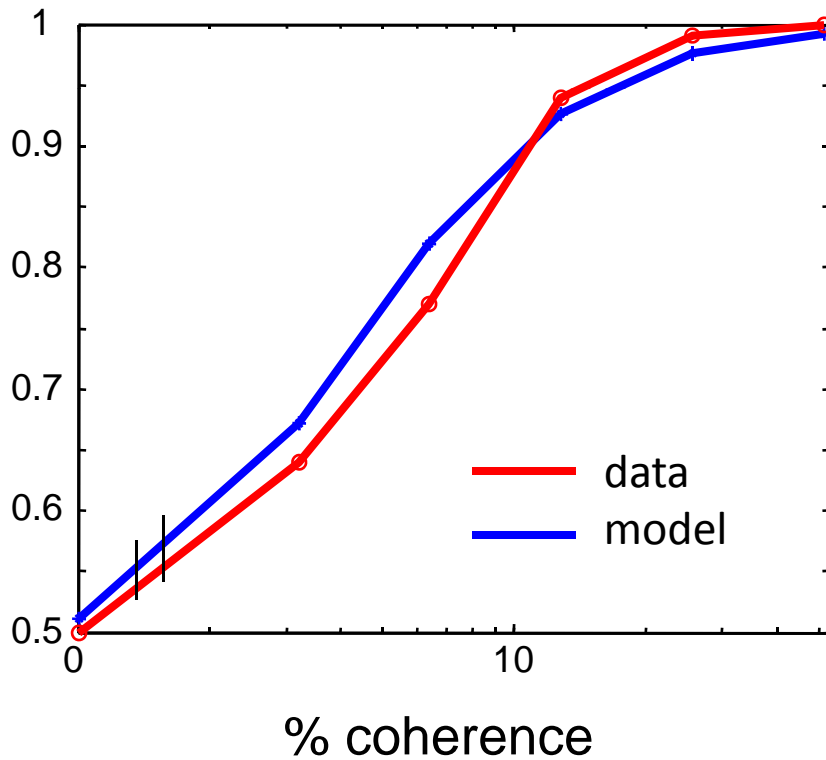
Maximum-likelihood estimate

Action selection : attractor dynamics

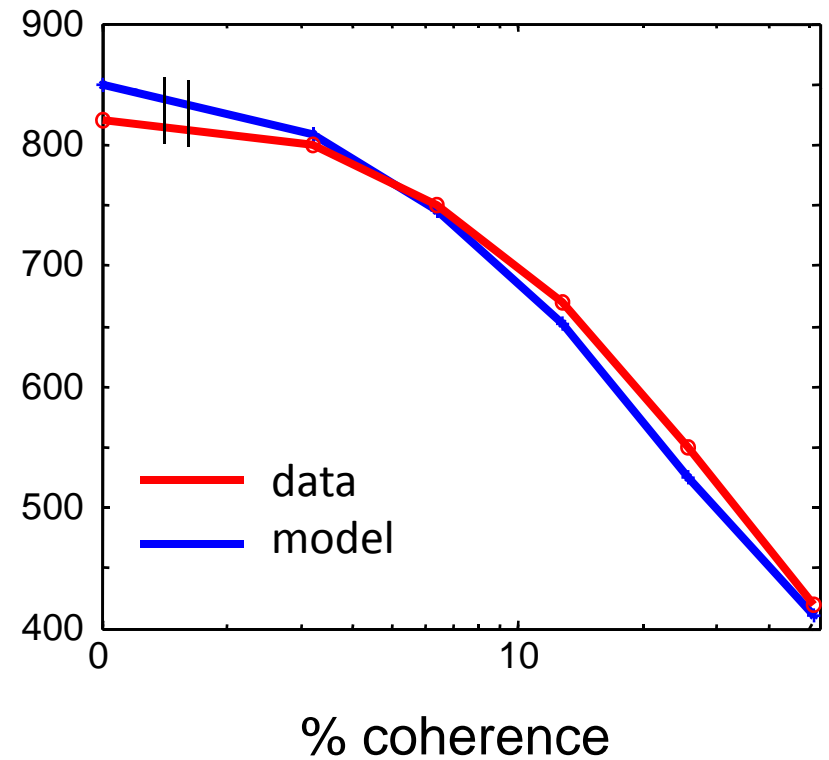


Binary Choice: Performance and Reaction Time

Probability correct



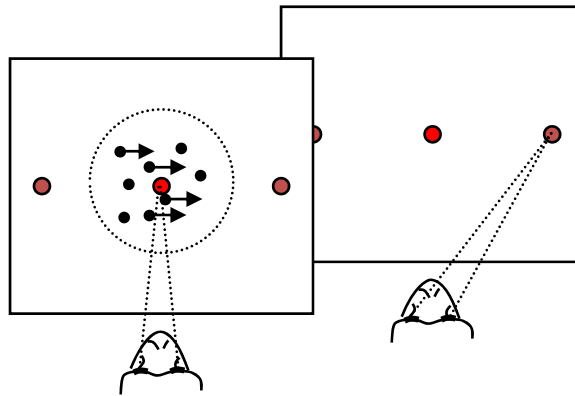
Reaction time (ms)



Data from Mazurek, Roitman, Ditterich, Shadlen, 2003

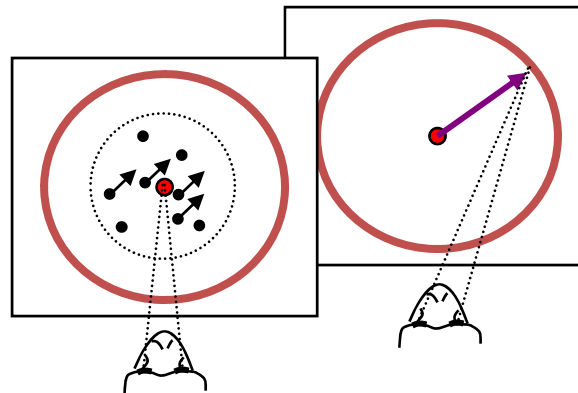
Beyond binary?

Binary



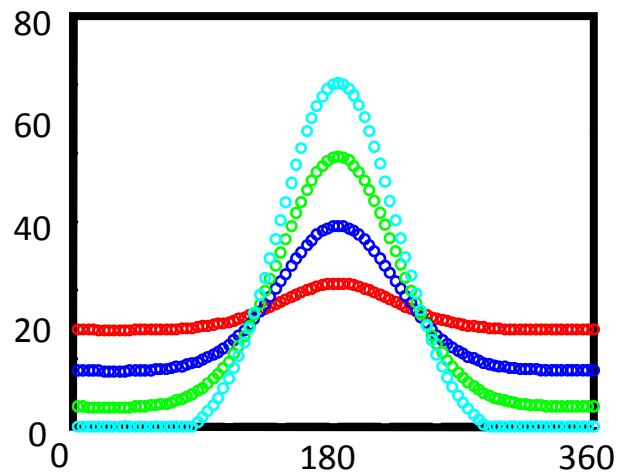
e.g. Roitman and Shadlen (2002)

Continuous



Continuous choice: Distribution encoded in LIP

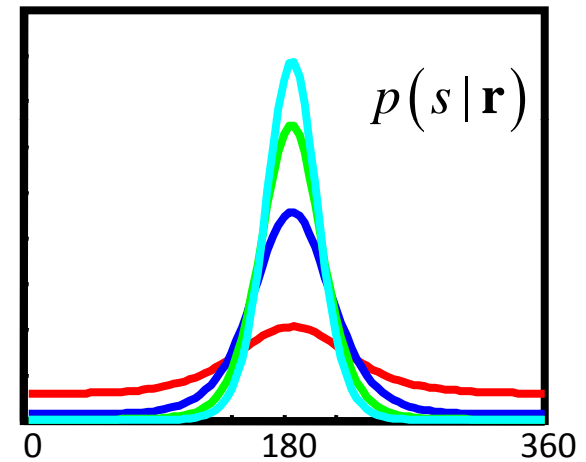
Firing rate (Hz)



Preferred saccade direction (°)

Probability

Bayes
→



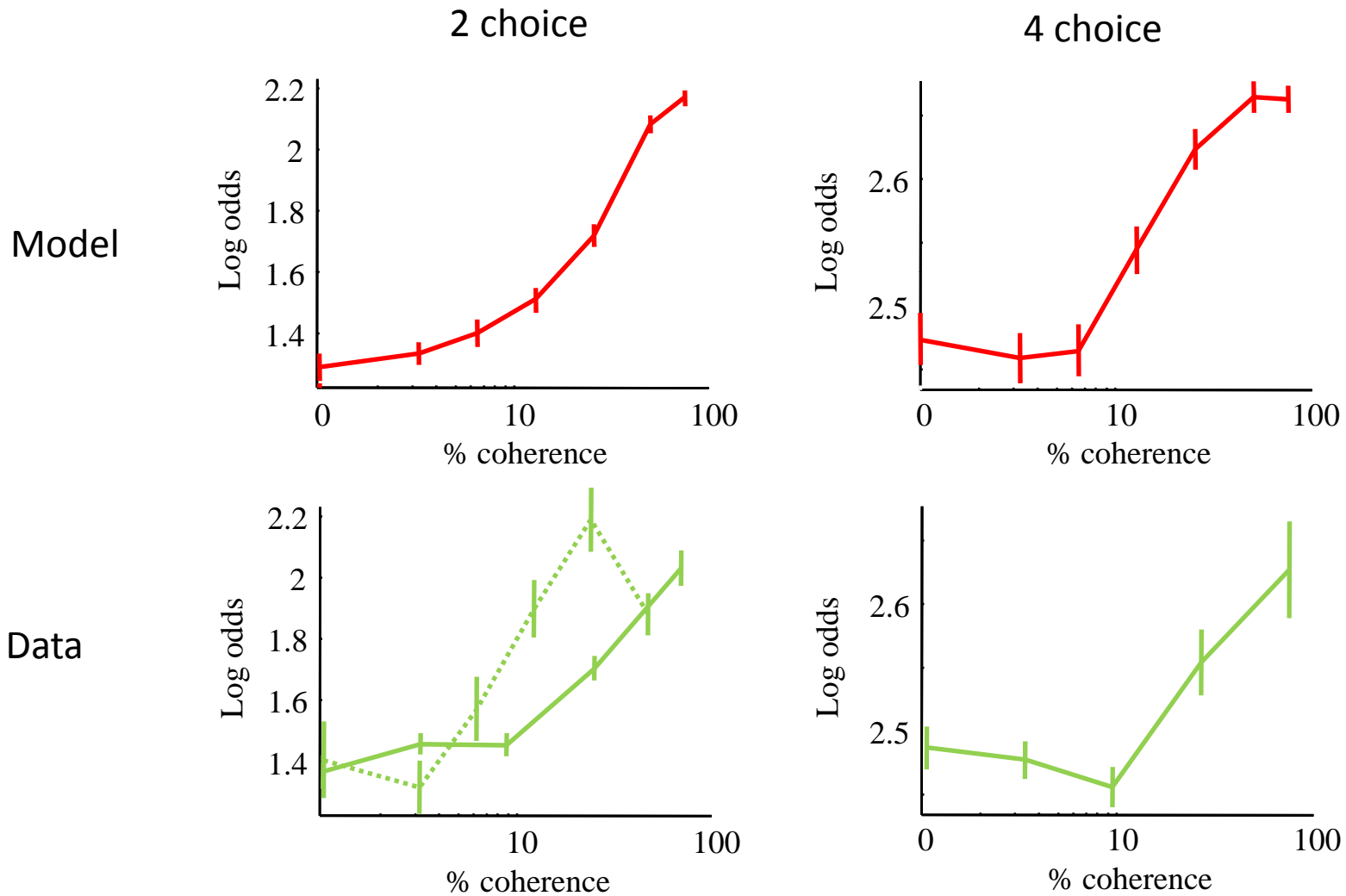
Saccade direction (°)

- 200 ms
- 150 ms
- 100 ms
- 50 ms

Experimental predictions

- Continuous case: width of population activity does not change over time
- LIP encodes a probability distribution over the stimulus. This distribution reflects both the reliability of the evidence and the performance of the animal.
- Same weights regardless of coherence or number of choices

Log odds at decision time

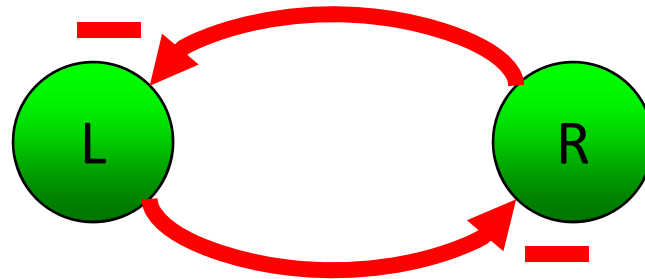


Data from Roitman and Shadlen (2002)

Data from Churchland et al. (2008)

The role of inhibition

- Bayesian model: Inhibition is needed only to keep neurons in their dynamical range.
- Other models: mutual inhibition essential.
- Open question



Conclusions

- Decision-making
 - Reward-based vs perceptual
 - Response time vs response signal
 - Binary vs multiple alternatives/continuous
 - Uncertainty can change over time or trials
- Diffusion models
- Accumulator models
- Bayesian models