CAAM/NEUR 415: EXAMINATION #2

December 2, 2013

1 Answer the following questions (total 28 points):

1. Describe the experimental protocol used to induce long-term synaptic potentation (LTP) discussed in the course. Explain how the synaptic specificity of the protocol can be tested. (3 points)

2. Is the variability observed in cortical pyramidal neuron spike trains consistent with integration of a large number (> 10) of synaptic inputs? Briefly justify your answer. Name one quantity used to characterize such variability and give a typical value for cortical pyramidal neurons in vivo. (3 points)

3. What is the minimum number of compartments needed to model the burst generation mechanism of CA3 pyramidal cells in the hippocampus? Briefly explain why a lower number will not work. (2 points)

4. You inject through an intracellular electrode a current $I = I_{max} \sin(2\pi\omega_1 t)$ of frequency ω_1 in a neuron that can be modeled as a single electrical compartment, with capacitance C and leak conductance g. What will the time course of the membrane potential depolarization be? Describe the time course qualitatively using words (hint: in what aspect will it be similar to the injected current and how will it differ from it?). If you simultaneously inject a second sinusoidal current $J = J_{max} \sin(2\pi\, 2\omega_1 t)$, at twice the frequency, $2\omega_1$, what will be the observed peak depolarization to $I + J$ in terms of those observed to I and J alone? (4 points)

5. Explain qualitatively using words the difference between the irradiance and illuminance of a light stimulus impinging on photoreceptors in the retina. Which one of these two quantities is most relevant to the nervous system and why? (3 points)

6. Which of the following two stimuli do you expect to generate more spikes in an ON-center retinal ganglion cell and why? 1) a bright spot in the center of the receptive field presented simultaneously with a bright spot in the surround; or, 2) the same bright spot in the center and a dark spot in the surround. (2 points)

7. What is the main difference between the spatial receptive field of simple and complex cells in visual cortex and retinal ganglion cells? How is this difference relevant in the context of natural stimuli? (2 points)

8. Which property of the spatio-temporal receptive field of simple or complex cells renders them directionally selective? Briefly explain why (use a sketch if necessary).

State the main difference between the spatial receptive field of simple and complex cells. (3 points)

9. Briefly explain in the context of the detection of weak light flashes by human observers, what the probability of false-alarm is. Briefly explain why it is important to monitor the probability of false-alarm in such experiments. (2 points)

10. Connect the following visual processing areas by arrows pointing in the direction of the natural sequence in which visual information successively propagates from the periphery towards more central areas: V1, Retina, MT and LGN. (2 points)

11. Explain in words what the autocorrelation function of a stochastic process tells you about it. What is the autocorrelation function of white noise? (2 points)

2 Theory and practice of Fourier transforms (total 26 points)

One of your friends claims that the Fourier transform of

$$g(t) = \frac{b}{b^2 + (a-t)^2} + \frac{b}{b^2 + (a+t)^2}$$

is given by

$$\hat{g}(\omega) = 2\pi e^{-b\,2\pi|\omega|} \cos(a\,2\pi\omega)$$

when $b = 0.1$ and $a = 2$. Check whether he/she is right by computing the Fourier transform numerically and comparing it to the exact formula as outlined below.

1. Plot the function $g(t)$ between -5.11 and $+5.12$ sec in an interval centered at zero using a time step $dt = 0.01$ sec and 1024 points. (2 points)

2. What is the range of frequencies (i.e., the Nyquist frequency $\omega_{Nyquist}$) that you will obtain and what is the resolution $d\omega$ that this data will give after a fast Fourier transform in the frequency domain? Briefly explain how you arrive at the results. (4 points)

3. Plot the predicted Fourier transform $\hat{g}(\omega)$ for the same range of frequencies between $-\omega_{Nyquist}$ and $\omega_{Nyquist}$ with a frequency step $d\omega$, as determined in 2. (3 points)

4. What do you need to take into account to compare the fast Fourier transform obtained using the MATLAB `fft` function with the theoretical formula for $\hat{g}(\omega)$ given above? Briefly explain. (5 points)

5. Directly compare the numerical Fourier transform and the exact predicted one. First, compute the fast Fourier transform using the `fft` function of MATLAB, and then take into account the factors determined in 4. Finally plot the real part of the numerical Fourier transform as a function of frequency between $-\omega_{Nyquist}$ and $\omega_{Nyquist}$ with a frequency step $d\omega$ and compare with the plot of 3.

Hints: Instead of multiplying by the phase factor derived in 4 you can also circularly shift the discretized finite samples g_j to start with the sample corresponding to $t = 0$ before carrying out the `fft`. In addition, don't forget to rearrange the frequency components in their natural order between $-\omega_{Nyquist}$ and $\omega_{Nyquist}$ after carrying out the `fft`. (3 points)

6. What is the maximal value of the imaginary part of the fast Fourier transform? Is it justified to neglect it? Explain why. (3 points)

7. If you had to verify the above relation between $g(t)$ and $\hat{g}(\omega)$ using only pencil and paper, how would you proceed, given the methods we have learned in the lecture to compute Fourier transforms? Can you briefly explain in words why the above relation seems correct? (6 points)

3 Effect of half-wave rectification and gain control on the responses of Gabor filters to moving gratings (total 18 points)

Assume that the response of a directionally selective simple cell to moving gratings is described by

$$R_{rect}(t) = R_{max} \frac{\lfloor R_1(t) \rfloor_+^2}{\sigma^2 + \lfloor R_1(t) \rfloor_+^2} \tag{1}$$

where

$$\lfloor R \rfloor_+ = \begin{cases} R & \text{if } R \geq 0, \\ 0 & \text{if } R < 0, \end{cases}$$

and

$$R_1(t) = R_{spont} + R_c(t) \quad \text{with} \quad R_c(t) = \int dx\, dt_0\, g_e^-(x, t_0) c(x, t - t_0).$$

The parameters are: $R_{spont} = 10$ spk/s, $\sigma = 60$ spk/s and $R_{max} = 120$ spk/s. The spatio-temporal receptive field is given by

$$g_e^-(x, t) = \frac{1}{2\pi\sigma_x\sigma_t} e^{-x^2/2\sigma_x^2 - t^2/2\sigma_t^2} \cos(2\pi k_x x - 2\pi k_t t)$$

with $\sigma_x = 0.1$ deg, $k_x = 4.2$ c/deg, $\sigma_t = 31$ msec and $k_t = 8$ c/sec.

1. Plot R_{rect} as a function of R_1. Use values of R_1 between 0 and 150 spk/s. (4 points)

2. Compute and plot the response $R_{rect}(t)$ for a grating given by $c(x, t) = \cos(2\pi\eta_x x \pm 2\pi\eta_t t)$ moving either to the left or to the right. Use the parameters $\eta_x = 4$ c/deg and $\eta_t = 8$ c/sec and a time interval of motion of 2000 ms. (14 points)

4 Detection of weak light flashes by retinal ganglion cells (total 28 points)

A model used to describe the discharge of retinal ganglion cells in response to weak light flashes assumes that photons are absorbed according to a Poisson process and filtered through an exponential low-pass filter,

$$f(t) = \begin{cases} Ce^{-t/\tau}, & t \geq 0, \\ 0, & t < 0, \end{cases}$$

with a time constant $\tau = 50$ ms. The resulting continous wave form is then used to drive an inhomogeneous Poisson process that represents the ganglion cell spike train. The constant C is chosen such that

$$\int_0^\infty f(t)\, dt = 3.$$

This implies that, on average, 3 spikes are generated per absorbed photon.

1. Generate a 500 ms long Poisson event train (each δ-function/event in the train represents the absorption of one photon) with a mean value of 7 absorbed photons per second. Convolve this sequence with $f(t)$ to obtain a rate, $r(t)$, and plot five samples of this resulting continuous waveform. (5 points)

   Hints: Use a time step $dt = 0.1$ ms. Make sure the time units are consistent across your calculation. To verify that your convolution and event train are properly normalized, convolve $f(t)$ with an event train consisting in a single δ-function/event. The resulting rate $r(t)$ integrated over time should be equal to 3.

2. Use the waveform $r(t)$ obtained in 1 to drive an inhomogeneous Poisson process. Compute and plot from 100 sample spike trains the corresponding distribution of spike number over the 500 ms period. (6 points)

   Hint: Use a time step $dt = 0.1$ ms. To make sure your simulations are correct, ask yourself the following question: How many spikes do you expect on average?

3. Compute the mean spike number and the Fano factor of the spike count distribution. (5 points)

4. How does the distribution compare to a Poisson distribution with the same mean number of spikes? Justify your answer. (5 points)

5. Assume that spontaneous activity is described by the same model but with a mean number of absorbed photons equal to 4 per second. Compute and plot the corresponding spike count distribution as in 2. Plot the ROC curve based on a spike

count threshold. Compute the minimum error of an observer based on a spike threshold, under the assumption that light flashes and blank trials are presented with a probability of 1/2. (7 points)

Hint: Use 10 equally sized bins between 0 and 20 spikes to compute the ROC curve.