CAAM/NEUR 415: EXAMINATION # 1
March 4, 2003

Instructions:

(1) Time limit: 3 Hours.

(2) No outside sources.

(3) Work each problem in great detail. There are 3 problems.

(4) `Print your name on the line below`

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(5) Indicate your compliance with the honor system by writing out in full and signing the traditional pledge on the lines below

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(6) **Staple** this cover sheet to your solutions and return it to me by 5 pm on Tuesday, March 18.

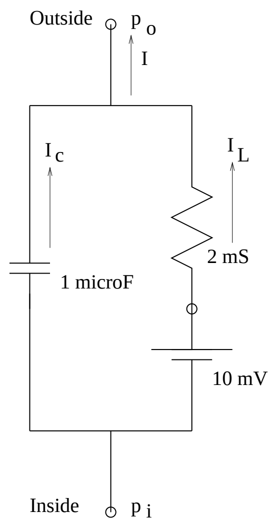

Figure 1.

1. With respect to the circuit of figure 1:
   (a) Derive a first order differential equation for the transmembrane potential, $V = p_i - p_o$.
   (b) Solve this equation subject to the initial condition $V(0) = 5\ mV$.
   (c) Write out the forward Euler discretization of the equation in (a). Establish a condition on the timestep, dt, for which it is stable.
   (d) Write out the backward Euler discretization of the equation in (a). Show that it is stable regardless of the size of dt.
   (e) If this were a postsynaptic patch we might replace the static conductance with something like $g(t) = t\exp(-t)$. Please write out and solve the corresponding differential equation, still subject to $V(0) = 5\ mV$.

2. The most simple active patch is the one with FitzHugh-Nagumo dynamics.

$$v' = 100v(v - 0.1)(1 - v) - 100n$$
$$n' = v - n/2$$

   (a) In the (v, n) phase-plane, where $-.5 \le v \le 1.2$ and $-.1 \le n \le .6$ please sketch the v and n nullclines.
   (b) In the same phase-plane please sketch three representative solution trajectories. One does this by following the arrows. The arrows come from simple substitution. For example, at $v = n = 0.1$ there is an arrow pointing in the direction $(100 * 0.1 * (0.1 - 0.1) * (1 - 0.1) - 100 * 0.1, 0.1 - 0.1/2)$, i.e., in the direction $(10, .05)$.
   (c) Write out a hybrid Euler discretization of the FitzHugh-Nagumo system.

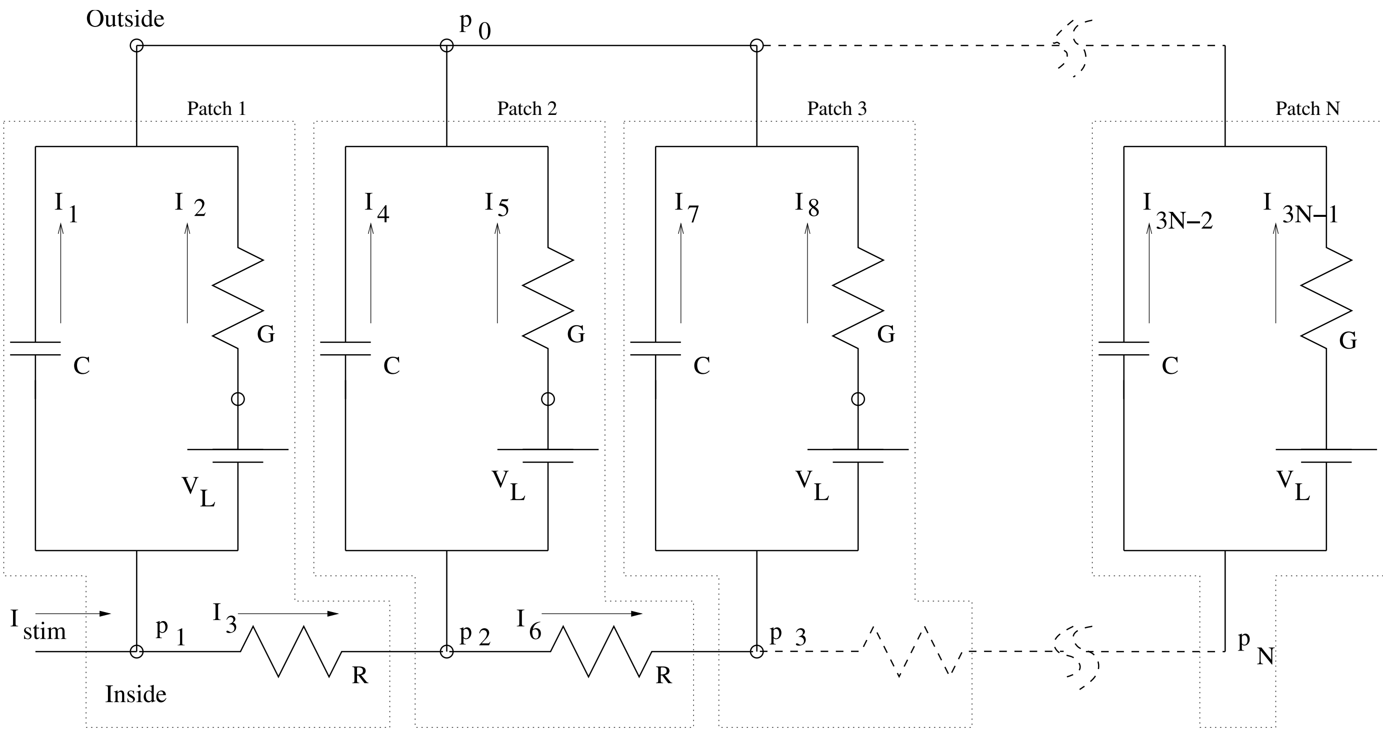

Figure 2.

3. With respect to the circuit in figure 2:

(a) Derive a linear system of first order differential equations for the $v_j = p_j - p_o - V_L$. Please show all of your work. Express your final answer in matrix-vector notation.

(b) Write out the forward Euler discretization of the equation in (a). Establish a condition on the timestep, dt, for which it is stable.

(c) Write out the backward Euler discretization of the equation in (a). Show that it is stable regardless of the size of dt.