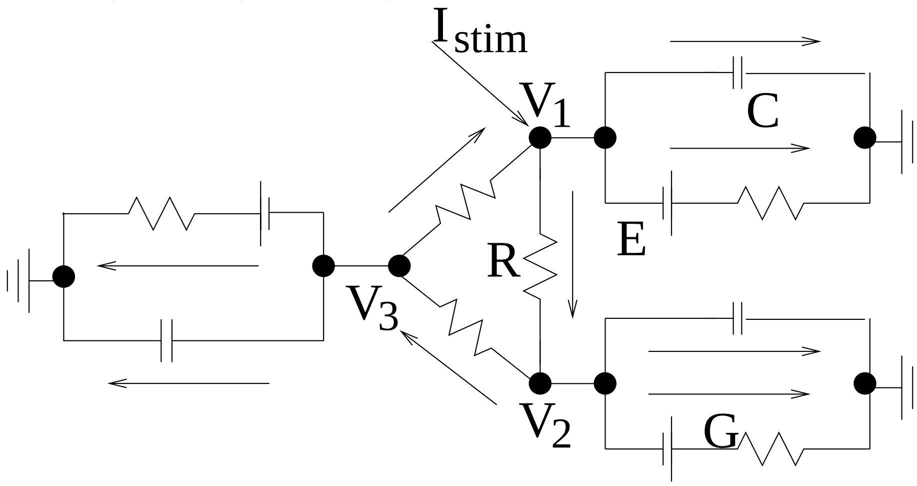
CAAM/NEUR 415, Exam 1, Fall 2008, Nine Problems



1. [10 points] Please interpret, in full sentences, the circuit above as three coupled single compartment cells. (Each capacitance is C, each battery is E, the conductance in series with each battery is G, and the remaining resistors each have value R).

2. [**10**] Carefully derive three coupled ordinary differential equations for V_1, V_2 and V_3.

3. [**10**] Define $v_j = V_j - E$, $j = 1, 2, 3$, and $v = [v_1\ v_2\ v_3]^T$ and show that v obeys

$$v'(t) = Bv(t) + f(t), \quad v(0) = 0. \tag{1}$$

Express the solution to this equation in terms of the (yet to be determined) eigenvalues and (normalized) eigenvectors of B.

4. [**10**] Consider B when $C = R = G = 1$. Show that -1, -4 and -4 are the eigenvalues of B. and compute their associated eigenvectors. When normalizing please do not approximate any square roots. Hint: one eigenvector is

$$\frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

and the other two may chosen so that one has a zero in its second component and the other has a zero in the third.

5. [**10**] Insert these into the general solution found in (3) and explicitly evaluate $v(t)$ when

$$I_{stim}(t) = 18(\exp(-t) - \exp(-2t))$$

Hint: $v_2(t) = v_3(t)$ and

$$v(t) = 6\exp(-t)(t - 1 + \exp(-t))\begin{pmatrix}1\\1\\1\end{pmatrix} + \ldots$$

6. **[10]** If we now add a simple ion channel to each membrane compartment we find at node one that current balance requires (retaining $C = R = G = 1$)

$$V_1'(t) + (V_1(t) - E) + 2V_1(t) - V_2(t) - V_3(t) + g_m m_1(t)(V_1(t) - E_m) = I_{stim}(t)$$

where m_1 is presumed to be governed by

$$m_1'(t) = m_\infty(v_1(t)) - m_1(t).$$

Please explain, in full sentences, the biological significance of g_m, m_1 and E_m and their role in these two equations.

7. **[10]** Please derive the current balance and gating equations at the other two nodes. What equation must the rest potential, call it $\overline{V}$, obey?

8. **[10]** Suppose that $I_{stim} = \varepsilon I_0$ and assume that

$$V_j(t) = \overline{V} + \varepsilon \tilde{v}_j(t) + \ldots \quad \text{and} \quad m_j(t) = m_\infty(\overline{V}) + \varepsilon \tilde{m}_j(t) + \ldots$$

Show that

$$\tilde{v} = \begin{pmatrix} \tilde{v}_1 \\ \tilde{v}_2 \\ \tilde{v}_3 \end{pmatrix} \quad \text{and} \quad \tilde{m} = \begin{pmatrix} \tilde{m}_1 \\ \tilde{m}_2 \\ \tilde{m}_3 \end{pmatrix}$$

together are governed by a linear system of the form

$$\begin{pmatrix} \tilde{v}' \\ \tilde{m}' \end{pmatrix} = \begin{pmatrix} B - aI & bI \\ cI & -I \end{pmatrix} \begin{pmatrix} \tilde{v} \\ \tilde{m} \end{pmatrix} + \begin{pmatrix} I_0 \mathbf{e}_1 \\ 0 \end{pmatrix}$$

where B is the 3-by-3 matrix derived in questions 3 and 4 and I is the 3-by-3 identity matrix. Please carefully identify the scalars a, b and c.

9. **[10]** Let us now study the eigenvalues and eigenvectors of

$$Q \equiv \begin{pmatrix} B - aI & bI \\ cI & -I \end{pmatrix}$$

In particular, from

$$\begin{pmatrix} B - aI & bI \\ cI & -I \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \lambda \begin{pmatrix} p \\ q \end{pmatrix}$$

deduce that q is an eigenvector of B, that $p = (\lambda + 1)q/c$ and that λ is determined by

$$\lambda - \frac{bc}{\lambda + 1} + a = z$$

where z is an eigenvalue of B. Write down all 6 eigenvalues and their associated eigenvectors in the case that $a = 2$, $b = 4$ and $c = 6$.