CAAM/NEUR 415: EXAMINATION # 1

March 6, 2005

1. Let us consider the equations of Hodgkin and Huxley on a patch.

$$\begin{aligned} C_m v'(t) &= -G_L(v-E_L) - G_{Na}m^3h(v-E_{Na}) - G_K n^4(v-E_K) \\ m'(t) &= (1-m)\alpha_m(v) - m\beta_m(v) \\ h'(t) &= (1-h)\alpha_h(v) - h\beta_h(v) \\ n'(t) &= (1-n)\alpha_n(v) - n\beta_n(v) \end{aligned} \tag{1}$$

(a) [10 points] Define and explain the significance of each term in this system.

(b) [10 points] Explain the origin of each of these equations. More precisely, give the balance law and/or phenomenological basis for each equation.

(c) [10 points] By what experimental means are the six rate functions determined?

(d) [12 points] We investigated the stability of the rest solution of a reduced version of (1) via phase plane techniques. Although such graphical means are not applicable to the full system it still makes sense to investigate the eigenvalues of the Jacobian, J, at rest. The rest state is

$$v = 0, \qquad m = m_0 \equiv \frac{\alpha_m(0)}{\alpha_m(0)+\beta_m(0)},$$

$$h = h_0 \equiv \frac{\alpha_h(0)}{\alpha_h(0)+\beta_h(0)}, \qquad n = n_0 \equiv \frac{\alpha_n(0)}{\alpha_n(0)+\beta_n(0)}.$$

In this case J is the 4-by-4 matrix who's ith row is composed of the four partial derivatives (with respect to v, m, h and n) of the right-hand side of the ith equation in (1), evaluated at rest. I offer up 4 of these terms in

$$J = \begin{pmatrix} -G_L - G_{Na}m_0^3h_0 - G_K n_0^4 & G_{Na}3m_0^2h_0E_{Na} & J_{13} & J_{14} \\ (1-m_0)\alpha_m'(0) - m_0\beta_m'(0) & -\alpha_m(0) - \beta_m(0) & J_{23} & J_{24} \\ J_{31} & J_{32} & J_{33} & J_{34} \\ J_{41} & J_{42} & J_{43} & J_{44} \end{pmatrix}$$

Please fill in the 12 missing terms.

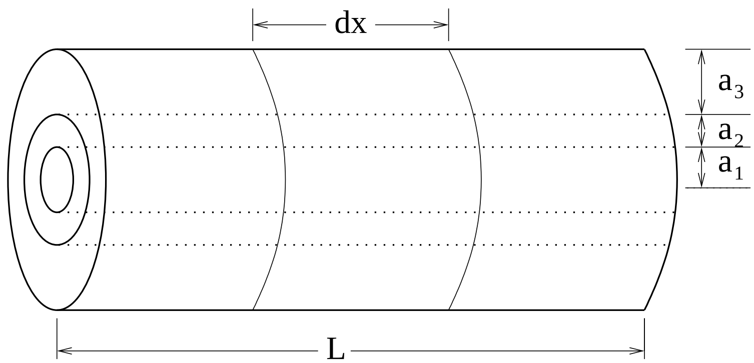
(e) [8 points] On using our standard numerical values for the conductances, reversal potentials and the like we find that the eigenvalues of J are roots of the quartic

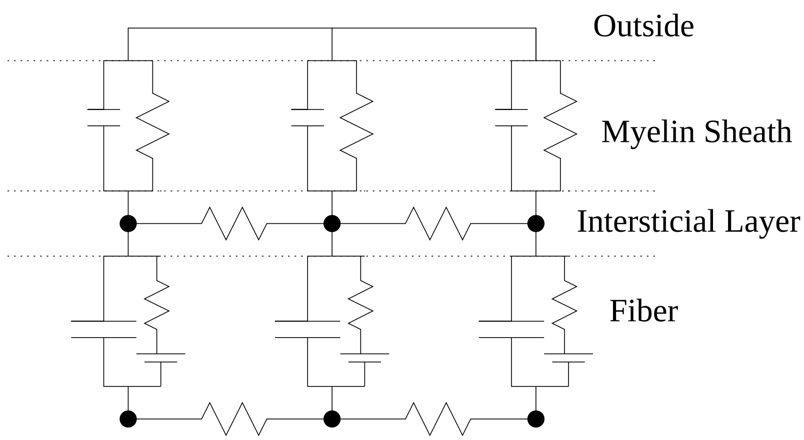
$$Q(z) = z^4 + 5.2z^3 + 2.7z^2 + 1.1z + 0.11$$

Where must the roots of Q lie for the rest state to be deemed stable? What can you tell me about the roots of this particular Q?

2. There is growing evidence that significant current may be flowing in the small intersticial layer between the axon fiber and its myelin sheath. Following the figure below let us build a simple passive compartmental model.

L cm is the length of the myelinated axon. **dx cm** is the length of each comparment. The fiber radius is $\mathbf{a_1}$ **cm**, its intracellular resistivity is $\mathbf{R_2}$ **kΩcm**, its membrane capacitance is $\mathbf{C_m}$ $\boldsymbol{\mu}\mathbf{F/cm^2}$, its membrane conductance is $\mathbf{G_m}$ $\mathbf{mS/cm^2}$, and its membrane rest potential is $\mathbf{E_r}$ **mV**. The interstitial layer is an annulus of thickness $\mathbf{a_2}$ and its intracellular resistivity is $\mathbf{R_i}$ **kΩcm**. The myelin sheath is an annulus of thickness $\mathbf{a_3}$, its sheath capacitance is $\mathbf{C_s}$ $\boldsymbol{\mu}\mathbf{F/cm^2}$ and its sheath conductance is $\mathbf{G_s}$ $\mathbf{mS/cm^2}$.





(a) [15 pts] Scale these passive parameters by the proper patch lengths and areas to arrive at true resistances, conductances and capacitances and so label the circuit diagram with these symbols. For example, the true fiber patch resistance is $R_f \equiv R_2 dx/(\pi a_1^2)$.

(b) [20 pts] Label black dots as unknown potentials (v_1, v_2, v_3 in the fiber and w_1, w_2, w_3 in the intersticial layer) and label each edge with an arrow and unknown current. Carefully write down current balance at each of the black dots. Carefully express each of these balance equations in terms of the unknown potentials.

(c) [15 pts] Express these equations in matrix form with the unknowns ordered as $[v_1\ v_2\ v_3\ w_1\ w_2\ w_3]$. Generalize to the N compartment case $L = Ndx$, that is, express the $2N$ equations in matrix form. Pass to the continuum limit as $dx \to 0$ and $N \to \infty$ and derive a coupled system of partial differential equations (with boundary conditions) for v and w.