Instructions. Solve the following exercise. Return both the code written for MATLAB and figures, properly labeled. Answer the questions and justify your claims with clear and logical explanations.

1 Properties of poisson and gamma distributions

1. Plot on a single figure 10 spike trains, each 1 sec long, drawn from a Poisson process with mean rate 40 spk/sec. Use a temporal resolution of 1 ms for the spike timing.

2. Compute the variance in the spike count, $V(T)$ as a function of the mean spike count $N(T)$ for a gamma renewal process of order 2 with a mean rate of 40 spk/sec. Simulate 1000 random spike trains one second long. Then compute $N(T)$ and $V(T)$ on intervals going from 0 to 50 ms, 0 to 100 ms, ... up to 0 to 1 s, in steps of 50 ms. Compare the curve obtained in this manner with the theoretical formula,
$$N(T) = \frac{T}{m_{\Delta t}}, \quad V(T) = \frac{T}{2m_{\Delta t}} + \frac{1}{8}(1 - e^{-T/m_{\Delta t}}),$$
where $m_{\Delta t}$ is the mean inter-spike interval.

3. Compute the interspike interval correlation coefficient at lag 1 for a Poisson process of mean rate 40 spk/sec and an absolute refractory period of 2 msec. Use 1000 random interspike intervals.

4. Use the same interspike interval sequence Δt_i of the previous exercise to generate a new sequence of interspike intervals $\Delta t'_i$ as follows: $\Delta t'_1 = \Delta t_1$ and
$$\Delta t'_i = \begin{cases} \Delta t_i + 5 & \text{if } \Delta t_{i-1} < m_{\Delta t}, \\ \Delta t_i - 5 & \text{if } \Delta t_{i-1} \geq m_{\Delta t}, \end{cases}$$
for $i > 1$ and where $m_{\Delta t}$ is the mean interspike interval of the original sequence. Compute the correlation coeffficient at lag 1. Explain the results obtained here and in the previous exercise.

Hints. Use the matlab functions `exprnd, gamrnd, cumsum` to generate random intervals and their corresponding spike times. For problem 1, use the function `line([`t_{spk} t_{spk}`]',[0 1]')` to plot a tick mark for each spike at time t_{spk} on your plot. Read the MATLAB help to see how the call to `line` can be generalized to a n_{spk} x 2 array of n_{spk} spikes.