CAAM 440 · APPLIED MATRIX ANALYSIS

Spring 2012 (2–2:50pm MWF)

Instructor: Mark Embree

The following outline is rough schedule for the semester; changes are inevitable.

Spectral Theory

1. Introduction: Review of basics; example from social networks
2. Resolvent and existence of eigenvalues; Schur triangular form
3. Spectral Theorem for Hermitian matrices; diagonalizable matrices
4. Application: optimally damped oscillator gives a nondiagonalizable matrix
   Introduction to the Jordan form (algebraic approach)
5. Jordan canonical form (algebraic approach)
6. Spectral Theorem: resolvent integrals (analytic approach)
7. Jordan form: synthesis of algebraic and analytic approaches

Hermitian matrices

8. Courant–Fischer minimax characterization of eigenvalues
9. Jacobi matrices: distinct eigenvalues, interlacing nodes
10. Rayleigh–Ritz eigenvalue approximation, interlacing
11. Application: eigenvalue avoidance in parameterized systems
12. Positive definite matrices
13. Commutation and similarity transformation
14. Application: saddle point systems in incompressible fluids, constrained optimization

Singular Value Decomposition

15. Singular value and polar decompositions
16. Moore–Penrose pseudoinverse and other pseudoinverses
17. Courant–Fischer minimax characterization of singular values; singular value inequalities
18. Application: facial identification via eigenfaces
19. Application: latent semantic indexing for data mining
20. Application: Principal Component Analysis in statistics

Tensors

21. Notation, basic algebraic properties
22. Low rank approximations; Tucker decomposition
23. Application: medical imaging

Geometry: inner products, norms

24. Positive definite matrices induce inner products, norms
25. Angles between subspaces; subspace intersections via the SVD
26. Adjoints; self-adjoint, normal matrices
    Application: energy norms in mechanical systems
27. Symmetric gauge functions and unitarily invariant norms; Schatten p-norms

Matrix perturbation theory

28. Rellich/Lidskii perturbation series; Puiseux series
29. Basic results for Hermitian, normal matrices; Bauer–Fike Theorem
30. Gerschgorin's Theorem; numerical range
31. Pseudospectra
32. Applications: sensitivity of polynomial roots; pole placement in control theory

Matrix functions

33. Definitions (via spectral interpolating polynomials; Cauchy integrals)
34. Matrix polynomials; Cayley–Hamilton Theorem
35. Examples: powers, exponential, other functions
36. Application: dynamical systems, transient behavior
37. Application: centrality ranking for social networks

Nonnegative matrices

38. Perron–Frobenius Theory
39. Perron–Frobenius Theory, continued
40. Stochastic matrices, Markov chains; PageRank

Ergodic theory

41. Von Neumann's mean ergodic theorem
42. Infinite products of matrices
    Application: spectral theory for discrete Schrödinger operators