CAAM 540 · APPLIED FUNCTIONAL ANALYSIS

Problem Set 2

Posted Wednesday 3 September 2008. Due Tuesday 9 September 2008.

Young, problems 2.5, 2.13; exercise 3.3; problem 3.9 [25 points each]

2.5 Prove that, for $\alpha \in \mathbb{D}$,

$$\{f \in RH^2 : f(\alpha) = 0\}$$

is a closed linear subspace of RH^2 (use Problem 1.11).

2.13 Prove that every finite-dimensional subspace of a normed space is closed.

3.3 Prove that ℓ^∞, the space of bounded sequences of complex numbers with the supremum norm is complete.

(ℓ^∞ denotes the complex vector space of all bounded sequences $x = (x_n)_1^\infty$ of complex numbers, with componentwise addition and scalar multiplication, and $\|x\|_\infty = \sup_{n\in\mathbb{N}} |x_n|$.)

3.9 Let E be the Banach space $\mathbb{R}^2$ with norm

$$\|(x_1, x_2)\| = \max\{|x_1|, |x_2|\}.$$

Show that E does not have the closest point property by finding infinitely many points in the closed convex set

$$A = \{(x_1, x_2) : x_1 \geq 1\}$$

which are at minimal distance from the origin.