

CAAM 551 Problem set 1

Due Weds. 2 Sep. 09.

Problem 1

For problem 1, please do not consult with other students and do not share code. You may consult a text book but you may not directly copy any code.

Write a matlab code that will accept the output from LUfacC and produce a numerical solution x to the equation $Ax = b$.

The code LUfacC.m may be found on the CAAM 551 web page

<http://www.caam.rice.edu/~caam551/>

under Matlab Code.

Test your code on a random example of order $n = 100$.

i.e.

```
n = 100;
A = randn(n); b = randn(n,1);

[A,p] = LUfacC(A);

x = Solve(A,p,b);
```

Your code should satisfy the requirements of the documentation listed below. You will need to write the forward and backward triangular solves in order to construct the routine `Solve`. Both of these triangular solvers should be **column oriented**. (meaning array elements should be accessed sequentially down the columns and not across rows)

Your code should be documented at a level comparable to the documentation in LUfacC.m.

You should write a test code to verify that your answer is correct. The output of this code should give the size of the linear system residual and also the relative error between your computed solution and the one you get with the matlab command $x = A \setminus b$.

Use norms to display these results! I do not want to see matrix or vector elements.

Repeat the above test with a succession of three progressively more ill-conditioned matrices (condition numbers $10^4, 10^8, 10^{12}$). You can create an ill conditioned matrix as follows. Note that for these tests, I want you to generate the right hand side as described in the code segment given below. You are to compare your computed solution with the known solution which will be the vector with all "ones" as components.

```
[U,R] = qr(randn(n)); [V,R] = qr(randn(n));

s = rand(n,1); s = sort(s);

d = < specify condition number here>;
%
%   choose d = 4, 8, 12
%
K = 10^d;

for j = 1:5, s(j) = s(n)/K; end;

A = U*diag(s)*V';

b = A*ones(n,1)
%
%   Please use this special right hand side. Now you know
%   the exact solution should be x1 = ones(n,1)
%   Compare your solution with this and also compute
%   the norm of the residual b - A*x
%   Report what you see
%
```

Code specification for Solve:

```
function [x] = Solve(A,p,b);
%
% Input:  A  an n by n matrix containing an LU decomposition of
%          a matrix A produced by LUfacC
%
%          p  an integer array of length n containing pivot
%          information.
%
%          b  a vector of length n
%
% Output: x  a vector of length n with Ax = b
%
```

Problem 2

Let A be an $n \times n$ symmetric positive definite matrix.

- Prove that MAM^T is symmetric and positive definite for any nonsingular matrix M of order n .

Note: MAM^T is called a congruence (transformation) of A .

- Let P be any permutation matrix of order n . Let

$$PAP^T = \begin{bmatrix} \alpha & a^T \\ a & \hat{A} \end{bmatrix}$$

Prove that $\alpha > 0$ and also that $A^{(2)} := \hat{A} - \frac{1}{\alpha}aa^T$ is symmetric and positive definite of order $n - 1$.

- Prove that every diagonal element of A is positive and also that $\max_{i,j} |A(i,j)| = A(k,k)$ for some k . In words, the maximum element occurs on the diagonal when A is positive definite.

Problem 3

Let A be an n by n nonsingular matrix. In the first step of the LU decomposition we first constructed a pivoting permutation between row 1 and another row to bring the element α of largest absolute value in the first column to the $(1,1)$ position and then we factored

$$P_1 A = \begin{bmatrix} 1 & 0 \\ \ell & I \end{bmatrix} \begin{bmatrix} \alpha & c^T \\ 0 & A^{(2)} \end{bmatrix}$$

Prove that $A^{(2)}$ is nonsingular without using determinants.