CAAM 552: HOMEWORK 3

Due at 5PM on April 21, 2017.
You may turn this homework in with no penalty by April 27, 2017.

(1) Consider the spatial discretization of the time-dependent heat equation
\[
\frac{\partial u}{\partial t} - \Delta u = 0, \quad x \in \Omega \\
u(x, 0) = u_0(x), \quad x \in \Omega \\
u(x, t) = 0, \quad x \in \partial \Omega.
\]
(a) Give a semi-discrete formulation for the symmetric and non-symmetric interior penalty DG methods.
(b) Derive semi-discrete energy estimates for both formulations in part (a).

(2) Consider the inviscid Burgers’ equation
\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0, \quad x \in [-1, 1]
\]
For smooth solutions, this equation yields the energy estimate
\[
\frac{1}{2} \frac{\partial}{\partial t} \|u\|_{L^2}^2 = \frac{u^3(-1)}{6} - \frac{u^3(1)}{6}.
\]
(a) Derive a discontinuous Galerkin method with central numerical fluxes such that the semi-discrete formulation yields a similar semi-discrete energy estimate.
(b) Derive a discontinuous Galerkin method with upwind numerical fluxes, and state the resulting semi-discrete energy estimate.