1. Prove that the DG method constructed in class for the acoustic wave equation is stable under periodic boundary conditions. Hint: multiply the DG system on the left by $p$ and $u$, sum the two equations up, and manipulate the integrals to cancel all spatial terms.

2. The code `advection_convergence.m` uses an upwind flux

$$\frac{a}{2}\|u\| - \tau \frac{|a|}{2}\|u\|, \quad \tau > 0$$

with $\tau = 1$. Modify the convergence code to run with a central flux $\frac{a}{2}\|u\|n$ (i.e. $\tau = 0$), and plot $L^2$ errors for $N = 1, \ldots, 4$. Describe how the error behaves differently compared to the $\tau = 1$ case.

3. The code computes only right hand sides of an ODE system

$$\frac{du}{dt} = -Au.$$  \hspace{1cm} (1)

It does so by efficiently evaluating the matrix-vector product $-Au$. Applying the right hand side to the canonical vectors $e_i$ results in the $i$th column of the matrix. Use this to build the global matrix form of the semi-discrete system for the periodic advection equation with $a = 1$. For $N = 4$ and $K = 16$, compute and plot the eigenvalues of $-A$ for $\tau = 0, .5, 1, 2$. You should see eigenvalues with negative real parts; since the exact solution to (1) is $u = e^{-At}$, this will imply that certain components of the solution are damped.

Based on the computed eigenvalues, how would increasing $\tau$ larger and larger affect the application of explicit time-steppers to (1)? (Hint: consider regions of stability).