Thesis Concept: Wavelet Transforms and Filter Banks

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The *discrete wavelet transform* decomposes a time series (or other sampled function) into l^2-orthogonal components each having a characteristic length scale. Actually there are many wavelet transforms, that is, many orthogonal bases of l^2 whose members are dilates and translates of a single "mother wavelet". Haar introduced the first such transform, around 1910. Beginning with Daubechies' pioneering work in the 1980's, many other wavelet transforms have been discovered.

Electrical engineers have also a natural interest in decomposition of time series according to scale. Longer scales pose less stringent sampling requirements, so such a scale decompositiion can lead to economical signal storage and processing. In the 1970's, several researchers in digital signal processing proposed *multirate filter banks* to accomplish scale-related decomposition and sampling.

This thesis will explain the close relation between these concepts, wavelet transform and filter bank, originating in two different disciplines. In fact, the discrete wavelet transform *is* a so-called conjugate mirror filter. In showing this equivalence, I will describe in detail the implementation of some special instances of the wavelet transform or filter bank. I will illustrate the decomposition of signals using the *WaveLab* software suite, written in Matlab by Donoho.

I will use as my initial sources *Introduction to Wavelets and Wavelet Transforms: A Primer* by Burrus, Gopinath, and Guo, *A Wavelet Tour of Signal Processing*, 2nd edition, by Mallat, and *Ten Lectures on Wavelets* by Daubechies. These texts give good overviews of the topic propsed for the thesis. Also their extensive bibliography should help me identify the primary literature.