

Lit Search Sketch: Sparse solutions of inverse problems by L^1 regularization

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Abstract

The “lit search sketch”, or initial literature search, should identify key references and place them within the context of the thesis. Especially important are works which directly address the thesis or claim. This strawman lit search sketch imagines a thesis recounting the development of the idea described in the title. That is, it’s an expository thesis, for which the lit search is a large part of the writing. However a lit search for a paper reporting original research would exhibit the same structure: background references defining the context of the thesis, key papers directly addressing, supporting, or setting the stage for the paper’s claim, and additional works addressing aspects besides the principal claim.

1 Background Work

Claerbout and Muir (1973) suggested that minimization of the l^1 norm amongst all possible solutions of a linear inverse problem might single out the sparsest solution. Other application of this idea to the same problem (sparse deconvolution) include (Levy and Fullagar, 1981; Oldenburg et al., 1983). The “ L^1 minimizer implies simplicity” idea has entered the vernacular of inverse and ill posed problems (see eg. (Dobson and Vogel, 1997; Acar and Vogel, 1994; Dobson and Santosa, 1994), as has its close cousin, total variation regularization. However until the late 1980’s no one really had a clue why it worked. [Here and in other sections discuss what each paper does, where it fits in the grand scheme of things - at more length than this!]

2 Key References

Santosa and Symes (1987) were the first to give a coherent mathematical argument, justifying the Claerbout-Muir hypothesis for the discrete deconvolution problem. This brilliant work gave sharp conditions under which the l^1 minimizer is the sparsest solution. (Donoho and Stark, 1989) carried this idea further in various ways. [blah blah].

3 Additional References

Recent work elaborates on this theme. [blah de blah blah] (Donoho and Elad, 2003; Daubechies et al., 2004; Candes et al., 2004; Tropp, 2004).

4 References

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