

Chapter 1

Introduction

Optimal control problems (OCPs) governed by ordinary differential equations arise in a wide range of applications. One particular field where these optimal control problems are abundant is the aerospace industry. Aerospace engineers have been solving optimal control problems for trajectory optimization, spacecraft attitude control, jet thruster control, missile guidance and many other applications for decades. Methods for obtaining these solutions are almost as copious as the applications themselves.

A traditional approach to solving OCPs entails forming the optimality conditions directly, using the calculus of variations and Pontryagin's maximum principle [42], and then solving the resulting equations to obtain the solution to the optimal control problem. This is known as the *indirect* approach for solving OCPs. The references [9, 23, 41, 42, 45, 46] present just a small sample of the work that discusses or applies indirect methods for the solution of optimal control problems. In rare cases the solution can be obtained analytically from these optimality conditions, but in general, approximation methods are used to solve the problem numerically. The optimality conditions of these problems generally take the form of differential algebraic equations (DAEs) or boundary value problems (BVPs). The approximate solution to the OCP can be obtained by using a BVP solver. Many such methods exist. Perhaps the most popular methods are multiple shooting and collocation. The reader is encouraged to consult [3] for more information on these and other numerical methods for solving BVPs.

Alternatively, one can discretize the governing ODEs and the integral terms in the objective functional or constraint functions and thereby replace the infinite dimen-

sional optimal control problem by a large nonlinear programming problem (NLP). This is known as the *direct or direct transcription* approach for solving OCPs. This approach is typically easier to use, especially for OCPs with state equality or inequality constraints. Direct methods have been used, e.g., in [6, 7, 16, 44].

This thesis focuses on a class of direct transcription methods in which the governing ODEs are discretized using pseudospectral collocation methods. Such methods have attracted attention [15, 14, 17, 18, 20, 43] because of their alleged superior approximation properties and, in the case of Legendre pseudospectral method, the availability of a so-called adjoint map or estimate. However, most of the existing work in this area is numerical with incomplete, informal discussions of mathematical properties of pseudospectral discretizations for optimal control problems.

The goals of this thesis are to improve the mathematical understanding of pseudospectral discretizations for optimal control problems and to apply these methods to the solution of optimal control problems with significance to the aerospace community. In particular, we provide a systematic derivation of adjoint estimates for all pseudospectral discretizations that use Gauss-Lobatto points and we present rigorous results on the error between the solution computed using pseudospectral discretizations and the exact solution of the underlying infinite dimensional OCP.

Adjoint estimates provide approximations to the adjoint variables (also known as costate variables) corresponding to the optimal solution of the OCP in terms of the Lagrange multipliers corresponding to the NLP derived using the direct transcription method. Such approximations are important for error analysis, mesh refinement strategies, and real-time optimization using the method of neighboring extrema. Among the few results on adjoint estimates are [17, 26, 27]. In the context of pseudospectral discretizations, only [17] have provided an adjoint estimation procedure for the particular case of Legendre pseudospectral discretizations. This thesis provides a systematic derivation of adjoint estimates for all Gauss-Lobatto pseudospectral discretizations, which as a special case includes the result of [17]. The work on ad-

joint estimation provides the foundation towards a rigorous convergence analysis that provides estimates for the error between the solution of the infinite dimensional optimal control problem and associated adjoint as well as the solution of the discretized optimal control problem and associated Lagrange multipliers. Such error estimates are not available in the existing literature. This thesis derives error estimates for linear-quadratic optimal control problems, and presents numerical results for both linear-quadratic and nonlinear optimal control example problems.

In the second part of this thesis, a class of pseudospectral direct transcription methods are applied to a series of optimal control problems derived from the International Space Station (ISS) momentum dumping problem. This is an attitude control problem where the attitude of the station is manipulated by a controller which uses control moment gyroscopes (CMGs). The issue here is that the CMGs have a maximum momentum threshold which cannot be exceeded. Doing so will result in loss of control of the vehicle. The goal is to find a control trajectory that will maneuver the attitude of the ISS from some initial state to some final state with minimal total momentum on the CMGs, obeying the system dynamics and never exceeding the momentum threshold along the way. What makes this problem difficult is the severe nonlinearity of the problem and the possible discrete nature of the controls. Related spacecraft control problems are discussed in [1, 5, 8, 13, 36, 38, 43, 45]. This thesis includes a study of the numerical solution to the ISS momentum dumping problem which demonstrates the utility of pseudospectral methods for the direct transcription of optimal control problems.

This thesis is organized as follows. Chapter 2 states the general form of the optimal control problems that will be considered, their corresponding optimality conditions and provides some examples problems that will be used throughout this thesis. Chapter 3 states the optimality conditions of the discretized OCP, describes how adjoint estimates are obtained and explores some of the consequences of using pseudospectral methods in the direct transcription of optimal control problems. The

application of the Legendre pseudospectral method to the space station momentum dumping problem is addressed in Chapter 4. Finally, Chapter 5 contains remarks, conclusions and suggestions for future work.