1. [5 points] Solve \( x_{n+1} = x_n / 10 \) subject to \( x_0 = 100 \).

2. [5 points] Solve \( x_{n+1} = x_n / 10 + 9 / 10 \) subject to \( x_0 = 100 \).

3. Consider the pair of first order equations

\[
\begin{align*}
x_{n+1} &= 2x_n - y_n \\
y_{n+1} &= -x_n + 2y_n
\end{align*}
\]  

(i) [5 points] Show that \( x_n \) obeys the second order equation

\[ x_{n+2} = 4x_{n+1} - 3x_n \]  

(ii) [5 points] Insert the guess \( x_n = c\lambda^n \) into equation (2) and find the two roots \( \lambda_\pm \).

(iii) [5 points] The general solution is now \( x_n = c_+ \lambda_+^n + c_- \lambda_-^n \). Find \( c_\pm \) when \( x_0 = 1 \) and \( y_0 = 0 \).

(iv) [5 points] Write equation (1) as a matrix equation \( V_{n+1} = MV_n \). What is \( M \)?

(v) [5 points] Solve \( MV = \lambda V \) for the two eigenvalues, \( \lambda_\pm \), and associated eigenvectors, \( V_\pm \).

(vi) [5 points] The general solution is now \( V_n = c_+ \lambda_+^n V_+ + c_- \lambda_-^n V_- \). Find \( c_\pm \) when \( x_0 = 1 \) and \( y_0 = 0 \).

4. Suppose \( a, b \) and \( c \) are positive and consider

\[ x_{n+1} = f(x_n) \quad \text{where} \quad f(x) = \frac{cx}{1 + (ax)^b} \]

(i) [5 points] Graph \( f \) and find and label its two steady states. Under what condition on \( c \) are both such states nonnegative?

(ii) [5 points] Show that the nonzero steady state is stable so long as \( b < \frac{2c}{(c - 1)} \).

(iii) [5 points] Produce a careful cobweb sketch in the case that \( b = 1 \).
5. Suppose $a$, $b$, $c$ and $d$ are positive and consider

\begin{align*}
x_{n+1} &= ax_n - bx_n y_n \\
y_{n+1} &= dx_n y_n - cy_n
\end{align*}

(i) [5 points] Find the two steady states. Under what condition on $a$ are both such states nonnegative?

(ii) [5 points] Compute the matrix of partial derivatives of equation (3) and evaluate it at the nonzero steady state. Your final result should be

\[ J = \begin{pmatrix} 1 & -b(1 + c)/d \\ d(a - 1)/b & 1 \end{pmatrix} \]

(iii) [5 points] Compute the eigenvalues of $J$ and argue that at least of them lies outside the unit circle.