MATH 213 Spring 2008 Exam 2. 2 problems in 6 parts.

We will study the system

$$x' = x - x^3 - ay$$
$$y' = x - y$$

in two different regimes. In the first part we will assume $a > 1$ and in the second that $0 < a < 1$.

1. Let us assume that $a > 1$.

(i) [10 points] Calculate the x and y nullclines and draw the phase plane on large, carefully drawn axes. Evaluate the nullclines at many precise values of x, namely at

$$\{-3/2, -5/4, -1, -3/4, -1/2, -1/4, 0, 1/4, 1/2, 3/4, 1, 5/4, 3/2\},$$

and, for the purpose of graphing, you may assume the precise value $a = 2$. Carefully label the axes and nullclines.

(ii) [10 points] Calculate the direction of the flow on each nullcline on each side of every steady state assuming only that $a > 1$. Draw the associated arrows on the phase plane from part (i). Carefully justify every arrow.

(iii) [10 points] Compute the Jacobian and evaluate its eigenvalues at each steady state and assess the stability of each steady state assuming only that $a > 1$.

2. Let us assume that $0 < a < 1$.

(i) [10 points] Calculate the x and y nullclines and draw the phase plane on large, fresh, carefully drawn axes. Evaluate the nullclines at many precise values of x, namely at

$$\{-3/2, -5/4, -1, -3/4, -1/2, -1/4, 0, 1/4, 1/2, 3/4, 1, 5/4, 3/2\},$$

and, for the purpose of graphing, you may assume the precise value $a = 1/2$. Carefully label the axes and nullclines.

(ii) [30 points] Calculate the direction of the flow on each nullcline on each side of every steady state assuming only that $0 < a < 1$. Draw the associated arrows on the phase plane from part (2.i). Carefully justify every arrow.

(iii) [30 points] Compute the Jacobian and evaluate its eigenvalues at each steady state and assess the stability of each steady state assuming only that $0 < a < 1$.