**Numerical Methods for Ordinary Differential Equations**

We will develop and test three approaches on the simple planar system

$$\begin{aligned} x_1' &= x_1 - x_1^3 - a x_2 \\ x_2' &= x_1 - x_2 \end{aligned} \tag{1}$$

We begin with the canned solver `ode23`. In order to solve (1) from $t = 0$ up to $t = 20$ starting from $x_1(0) = x_2(0) = 1$ we type

```
[t,x] = ode23(@mycub,[0 20],[1 1]')
```

after creating an m-file called `mycub.m` that looks like,

```
function dx = mycub(t,x)
a = 2;
dx(1,1) = x(1) - x(1)^3 - a*x(2);
dx(2,1) = x(1) - x(2);
```

One may then examine $x$ in the phase plane via

`plot(x(:,1),x(:,2))` or `comet(x(:,1),x(:,2))`

or as natural time functions via

```
plot(t,x(:,1),t,x(:,2))
```

and in each case one should label the axes and curves via `xlabel`, `ylabel` and `legend`. One may now experiment with different time spans, initial conditions, and parameters (just $a$ for now). As a second example we encode the molecular switch in equation 29 on page 290 via

```
function dx = switch29(t,x)
dx(1,1) = x(1) - x(1)^2 - 2*x(1)*x(2);
dx(2,1) = x(2) - x(2)^2 - 2*x(1)*x(2);
```

To better appreciate what `ode23` is up to we now embark on our own approximation scheme. It is really just a step back to the early part of the course, for we will replace the differential equation, (1), with the difference equation

$$\begin{aligned} \frac{x_1(t_{n+1}) - x_1(t_n)}{dt} &= x_1(t_n) - x_1^3(t_n) - a x_2(t_n) \\ \frac{x_2(t_{n+1}) - x_2(t_n)}{dt} &= x_1(t_n) - x_2(t_n) \end{aligned} \tag{2}$$

where $dt$ is our time step, or increment, and $t_n = (n-1)dt$. A slight rearrangement of (2) reveals that we may solve for the state at $t_{n+1}$ knowing only the state at $t_n$. That is

$$\begin{aligned} x_1(t_{n+1}) &= x_1(t_n) + dt(x_1(t_n) - x_1^3(t_n) - ax_2(t_n)) \\ x_2(t_{n+1}) &= x_2(t_n) + dt(x_1(t_n) - x_2(t_n)) \end{aligned} \tag{3}$$

we have coded this in `mycub_fe.m`.