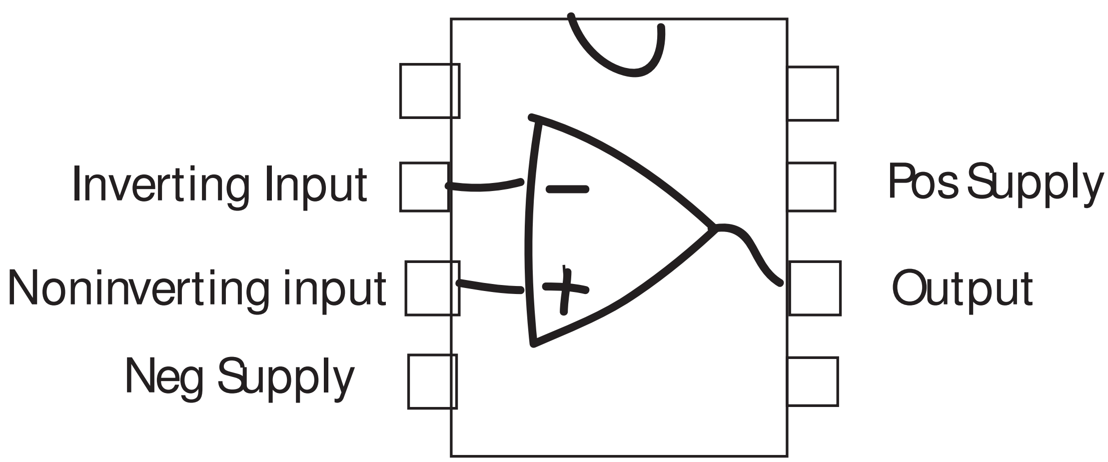
opamp

3. The Operational Amplifier

The operational amplifier, or op amp for short, is a wonderfully flexible electrical device. We will use them (in coming chapters) to both amplify and denoise neural signals as well as to mimic the complicated voltage–current relationship of the FitzHugh–Nagumo neuron. In this chapter we establish and demonstrate their basic behavior.

The basic op amp is a small complex circuit incased in a plastic chip with 8 leads and a small notch at one end. The notch helps us orient the chip and so connect the inputs and output to the proper terminals.



741

Figure 3.1. The op amp symbol and pin layout (for LM 741).

In circuit diagrams we presume the op amp is powered up and so we focus only on the $\pm$ input pins and the sole output. The key to designing and analysing op amp circuits lies in understanding its two basic laws,

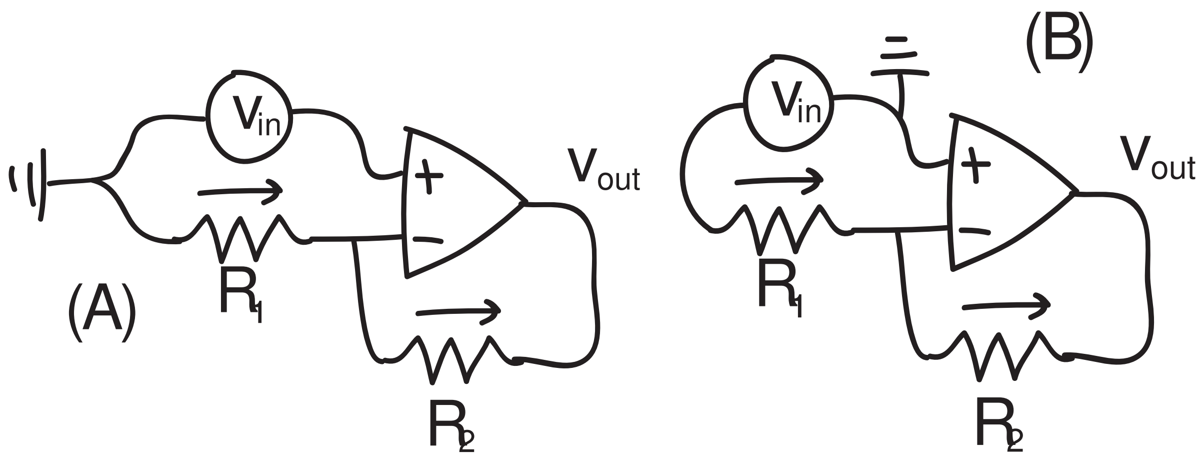
OA1: The potentials at the input pins coincide.

OA2: The current into each input pin is zero.

These two "laws" will permit us to calculate the **gain** of amplifier circuits and the **frequency** response of filter circuits.

oplay 3.1. The Algebra of Gain

We begin with the two simple amplifiers below.



op2

Figure 3.2. The noninverting, (A), and inverting, (B), amplifier circuits.

Regarding the noninverting circuit of Figure 3.2.A, we see from (OA1) that the potential at the minus pin is simply v_{in} and, from (OA2), that the two resistor currents must coincide. Ohm's law then permits us to conclude that

$$\frac{0 - v_{in}}{R_1} = \frac{v_{in} - v_{out}}{R_2}.$$

From here it is a simple matter to solve for

$$\boxed{\textbf{Figure } 3.2.\text{A:} \quad v_{out} = (1 + (R_2/R_1))v_{in}.} \qquad (3.1)_{\text{noninv}}$$

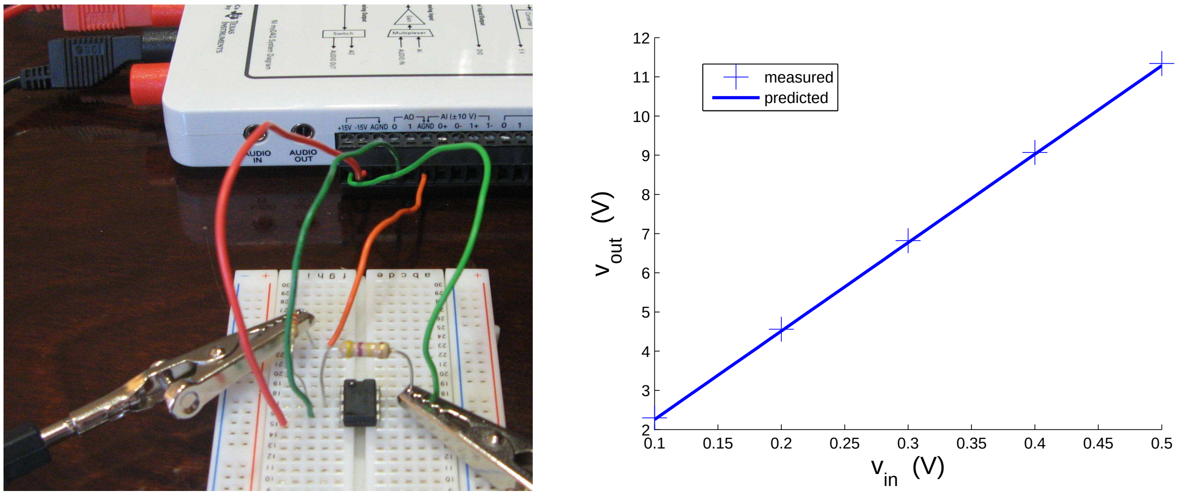
Regarding the inverting circuit of Figure 3.2.B, it follows from (OA1) that the potential at the minus pin is zero while (OA2) again permits us to equate the two resistor currents. In this case,

$$\frac{v_{in} - 0}{R_1} = \frac{0 - v_{out}}{R_2}$$

and so

$$\boxed{\textbf{Figure } 3.2.\text{B:} \quad v_{out} = -(R_2/R_1)v_{in}.} \qquad (3.2)_{\text{inv}}$$

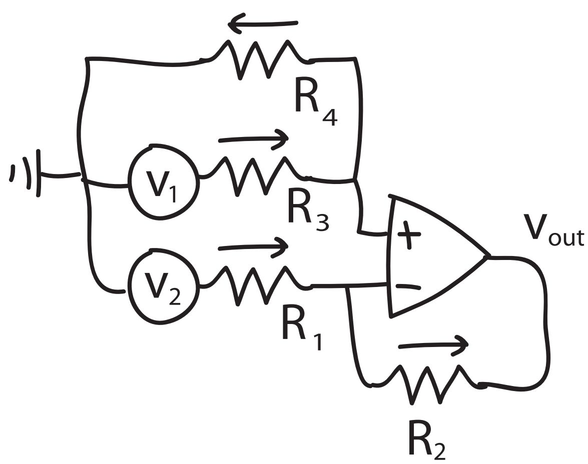
We examine how well this predicts observed behavior.



NInoninvert

Figure 3.3. A test of our theory. In the photo you see an opamp and two resistors, $R_1 = 21.7\ k\Omega$ and $R_2 = 468\ k\Omega$, in the noninverting amplifier configuration. The opamp is receiving power, $\pm 15\,V$, from the NI myDAQ card, into pins 4 and 7. The myDAQ also provides v_{in} into pin 3, and measures v_{out} via the alligator clips. We set the value of v_{in} in software (open NI ELVISmx Instrument Launcher, select the "Featured Instruments" tab and click on "DC Level Output") and measure the associated output by choosing the "DMM" instrument from the Launcher. We recorded v_{out} when v_{in} was set to 0.1, 0.2, 0.3, 0.4, and 0.5 Volts, and plotted our data (plus signs) against the gain formula (3.1).

We next consider a circuit that amplifies the **difference** between two input potentials, v_1 and v_2.



diffamp

Figure 3.4. The Differential Amplifier.

It follows from our ideal op-amp laws and KCL that the associated resistor currents obey

$$I_1 = I_2 \quad \text{and} \quad I_3 = I_4.$$

Ohm's law, together with $v_+ = v_- = v$ then yields

$$(v_2 - v)/R_1 = (v - v_{out})/R_2 \quad \text{and} \quad (v_1 - v)/R_3 = v/R_4.$$

We solve the latter for

$$v = \frac{R_4}{R_4 + R_3} v_1 \tag{3.3}$$ dav

and the former for

$$v_{out} = (1 + R_2/R_1)v - (R_2/R_1)v_2. \tag{3.4}$$ davo

On substituting (3.3) into (3.4) we find

$$\begin{aligned} v_{out} &= \frac{R_2}{R_1}\left(\left(\frac{R_1}{R_2} + 1\right)\frac{R_4}{R_4 + R_3} v_1 - v_2\right) \\ &= \frac{R_2}{R_1}\left(\frac{R_1 + R_2}{R_3 + R_4}\frac{R_4}{R_2} v_1 - v_2\right). \end{aligned} \tag{3.5}$$ davo1

We may turn this inner term into a simple difference if

$$\frac{R_1 + R_2}{R_3 + R_4}\frac{R_4}{R_2} = 1,$$

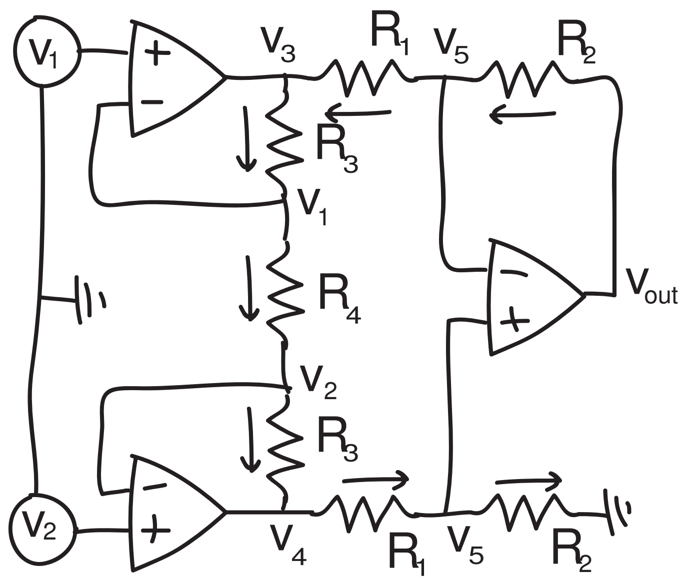
that is, if $R_1 R_4 = R_2 R_3$. A clean way to make this happen is to choose

$$R_1 = R_3 \quad \text{and} \quad R_2 = R_4 = G R_1,$$

for then (3.5) takes the simple form

$$\boxed{\textbf{Figure } 3.4: \quad v_{out} = G(v_1 - v_2).} \tag{3.6}$$ diffamp

In practice it is nice to be able to tune a single resistor to a desired gain. This is typically done though a circuit known as the Instrumentation Amplifier.



instamp

Figure 3.5. The Instrumentation Amplifier.

From OA1 we have determined the potential at either end of R_4 in terms of the two input potentials. Similarly, we have denoted the input potentials at the rightward opamp by v_5. We now use OA2 to work back from v_{out}.

To begin we note that the two upper horizontal currents must coincide. That is

$$\frac{v_{out} - v_5}{R_2} = \frac{v_5 - v_3}{R_1} \qquad (3.7)_{\text{iaup}}$$

Similarly, the two lower horizontal currents must also coincide. That is

$$\frac{v_5}{R_2} = \frac{v_4 - v_5}{R_1} \qquad (3.8)_{\text{ialow}}$$

We simplify by solving (3.7) for

$$\frac{v_5}{R_2} + \frac{v_5}{R_1} = \frac{v_4}{R_1}.$$

We then substitute this into (3.8) and find

$$v_{out} = (R_2/R_1)(v_4 - v_3). \qquad (3.9)_{\text{vo1}}$$

Next equating the top R_3 current and the R_4 current

$$\frac{v_3 - v_1}{R_3} = \frac{v_1 - v_2}{R_4}$$

and so

$$v_3 = v_1 + (R_3/R_4)(v_1 - v_2). \qquad (3.10)_{\text{iav3}}$$

Similarly, equating the bottom R_3 current and the R_4 current

$$\frac{v_2 - v_4}{R_3} = \frac{v_1 - v_2}{R_4}$$

brings

$$v_4 = v_2 + (R_3/R_4)(v_2 - v_1). \qquad (3.11)$$ iav4

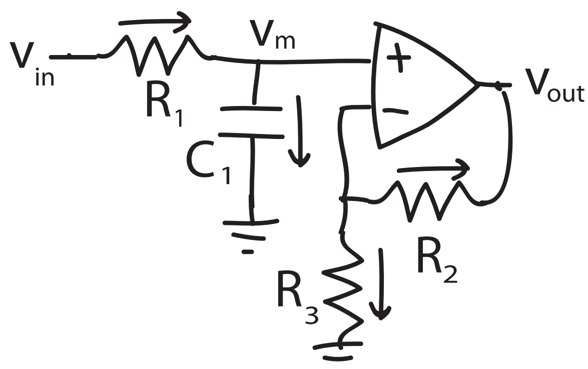
And so, on substituting (3.10) and (3.11) into (3.9) we find

Figure 3.5: $v_{out} = (2(R_3/R_4) + 1)(v_2 - v_1),$

and so recognize R_4 as our "variable" gain control.

filt 3.2. The Algebra of Frequency

We next add capacitors to our op amp circuits and investigate the associated transfer functions. We begin with the "first order filter" below.



fofil

Figure 3.6. A first order filter.

If we now balance currents at the two nodes we find

$$(v_{in} - v_m)/R_1 = C_1 v_m'$$
$$(v_m - v_{out})/R_2 = -v_m/R_3$$

where v_m denotes the unknown potential at each input terminal of the op amp. On substituting the first in the second we arrive at a differential equation for v_{out} in terms of v_{in}, (note that v_m has come and gone).

$$R_1 C_1 v_{out}'(t) + v_{out}(t) = (1 + R_2/R_3) v_{in}(t). \qquad (3.12)$$ fofilvo

If our input is of the form $v_{in}(t) \equiv V_{in}(s)\exp(st)$ then our output will take the form $v_{out}(t) = V_{out}(s)\exp(st)$ where V_{out} is determined by substituting these forms into (3.12). In particular

$$R_1 C_1 V_{out}(s) s \exp(st) + V_{out}(s)\exp(st) = (1 + R_2/R_3) V_{in}(s)\exp(st).$$

On canceling the common exponential and rearranging we arrive at the transfer function

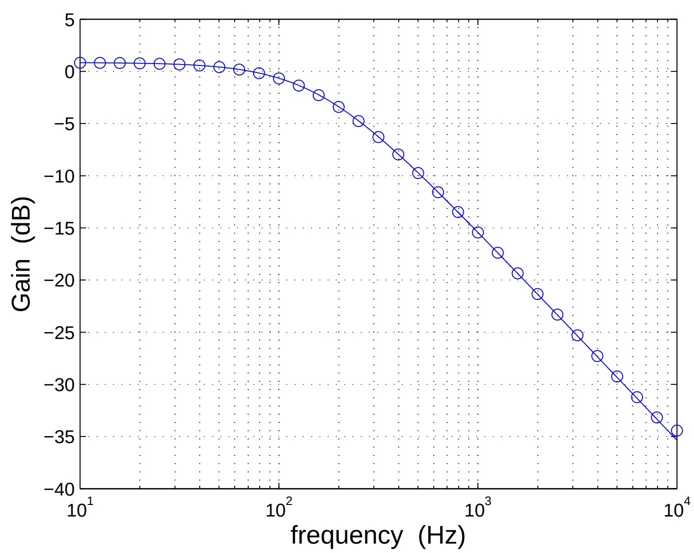
$$\boxed{\textbf{Figure } 3.6: \quad H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1 + R_2/R_3}{1 + sR_1C_1}}. \qquad (3.13)_{\text{fitrans}}$$

It is customary to represent this as

$$\text{Gain}(f) \equiv 20 \log_{10} |H(2\pi i f)|. \qquad (3.14)_{\text{gainf}}$$

We now examine *Gain* for the particular choice

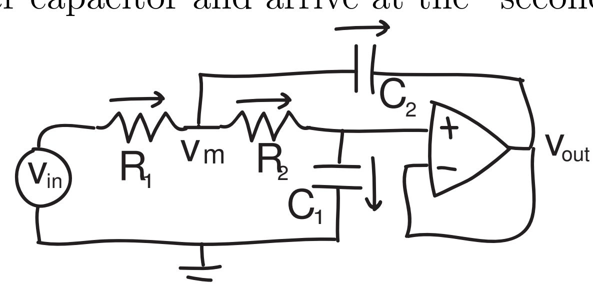
$$R_1 = 98.8\,k\Omega, \quad R_2 = 100.6\,k\Omega, \quad R_3 = 978\ k\Omega \quad \text{and} \quad C_1 = 10.4\ nF. \qquad (3.15)_{\text{fofilp}}$$



bodefofil

Figure 3.7. The Gain of the first order filter specified by (3.15). The solid line is a graph of (3.14). The circles are experimental data computed from the myDAQ Bode Analyzer.

We next add another capacitor and arrive at the "second order filter" below.



sofil

Figure 3.8. The second order filter.

Now balancing current at the only two interesting nodes reveals

$$(v_m - v_{out})/R_2 = C_1 v_{out}'$$
$$(v_{in} - v_m)/R_1 = (v_m - v_{out})/R_2 + C_2(v_m - v_{out})'.$$

On substituting the first in the second we arrive at a second order differential equation for v_{out} in terms of v_{in}, (v_m again has come and gone.)

$$v_{in} = R_1 C_1 R_2 C_2 v_{out}'' + C_1(R_1 + R_2) v_{out}' + v_{out}.$$

To find the associated transfer function we again suppose $v_{in}(t) = V_{in}(s)\exp(st)$ and $v_{out}(t) = V_{out}(s)\exp(st)$ and find

$$V_{in}(s) = (R_1 C_1 R_2 C_2 s^2 + C_1(R_1 + R_2)s + 1) V_{out}(s)$$

and so the transfer function is

Figure 3.8: $$H(s) = \frac{1}{R_1 C_1 R_2 C_2 s^2 + C_1(R_1 + R_2)s + 1}. \qquad (3.16)_{\text{sotrans}}$$

active

4. Building an Active Neuron

Our passive neuron was able to capture the response to small, subthreshold, stimulus, but had no ability to generate spikes. In this chapter we replace the chloride pathway with two pathways meant to derive from the action of potassium and sodium ions and achieve a neuronal model that spikes much like the true cell.

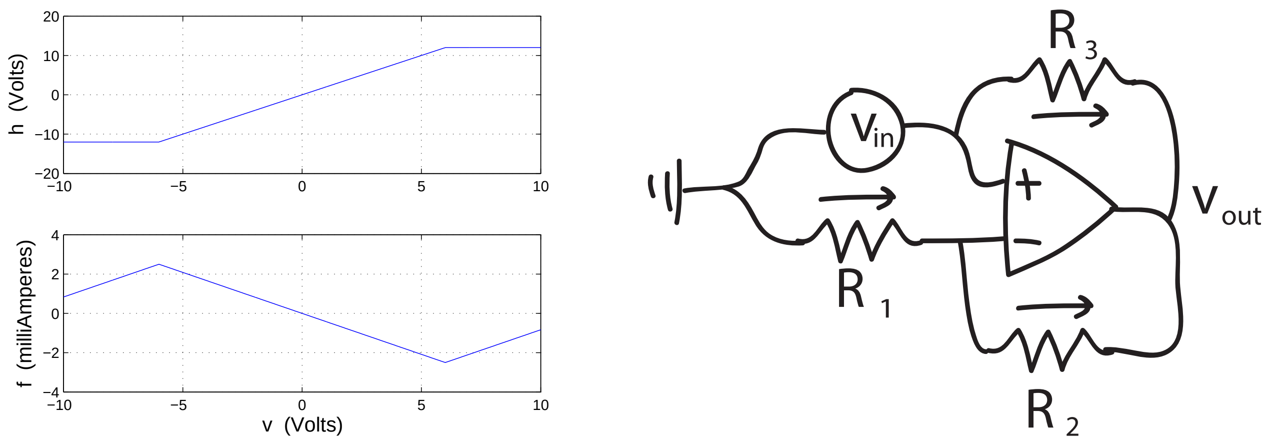
fnhard

4.1. In Hardware

We follow (Keener, 1983) and note that the noninverting amplifier of Fig. 3.2(A) obeys (3.1) only when v_{in} lies within bounds set by the supplied "rail" voltages, $\pm V_R$. In particular, if $R_1 = R_2$ then

$$v_{out} = h(v_{in}) = \begin{cases} -V_R & \text{if} \quad v_{in} < -V_R/2 \\ 2v_{in} & \text{if} \quad -V_R/2 < v_{in} < V_R/2 \\ V_R & \text{if} \quad V_R/2 < v_{in}. \end{cases}$$

We have graphed this "clipped" linear function in Figure 4.1(A).



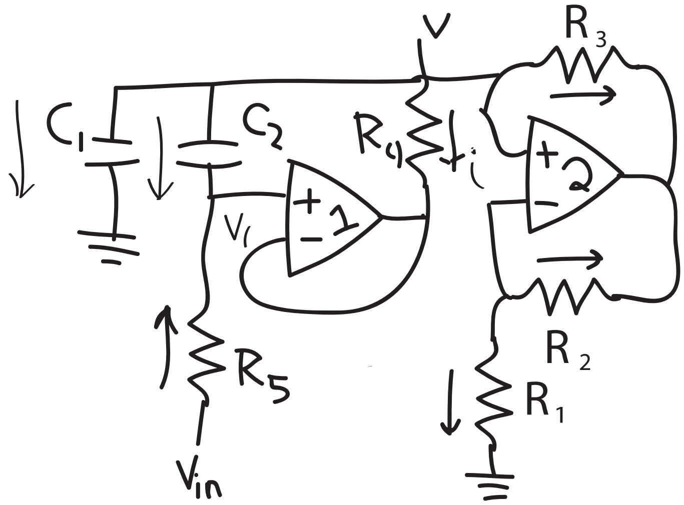
iNaIV

Figure 4.1. (A) The saturated gain of a simple ($R_1 = R_2$) noninverting amplifier with $V_R = 12$. (B) The current through R_3 in circuit (C) with $R_3 = 2.4\ k\Omega$. (C) The Keener analog of the sodium channel.

If we now add a third resistor as in Figure 4.1(C) it follows that the current through resistor R_3 is

$$I_3 = \frac{v_{in} - v_{out}}{R_3} = \frac{v_{in} - h(v_{in})}{R_3} = f(v_{in}) = \frac{1}{R_3} \begin{cases} v_{in} + V_R & \text{if} \quad v_{in} < -V_R/2 \\ -v_{in} & \text{if} \quad -V_R/2 < v_{in} < V_R/2 \\ v_{in} - V_R & \text{if} \quad V_R/2 < v_{in} \end{cases}$$

we have graphed this function in Figure 4.1(B). This nonlinear current–voltage device is an adequate approximation to the neuron's sodium current. We append to this a model of the potassium current and the capacitive current as depicted in Figure 4.2.



keener2

Figure 4.2. The Keener circuit analog of an active neuron.

We have labeled the one internal potential v_1. We now use KCL to derive a pair of equations for v and i (the current through R_4). Starting at the top we find

$$C_1 v' + C_2(v - v_1)' + i + i_3 = 0. \tag{4.1}$$ keen1

Next we balance the currents at v_1,

$$C_2(v - v_1)' = (v_1 - v_{in})/R_5 \tag{4.2}$$ keen2

and finally we express i via Ohm's Law,

$$i = (v - v_1)/R_4. \tag{4.3}$$ keen3

We manipulate (4.3) order to solve for the internal voltage

$$v_1 = v - R_4 i. \tag{4.4}$$ keen4

On plugging (4.4) into (4.2) and that into (4.1) we find

$$C_1 v' + (v - R_4 i - v_{in})/R_5 + i + f(v) = 0. \tag{4.5}$$ keen5

Next we differentiate (4.3) and plug into (4.2) and find

$$R_4 C_2 i' = C_2(v - v_1)' = (v_1 - v_{in})/R_5 = (v - R_4 i - v_{in})/R_5.$$

Minor manipulation then lands us at

$$\boxed{\begin{aligned} C_1 v' &= -f(v) - (1 - R_4/R_5)i - (v - v_{in})/R_5 \\ R_4 R_5 C_2 i' &= -R_4 i + v - v_{in}. \end{aligned}}$$

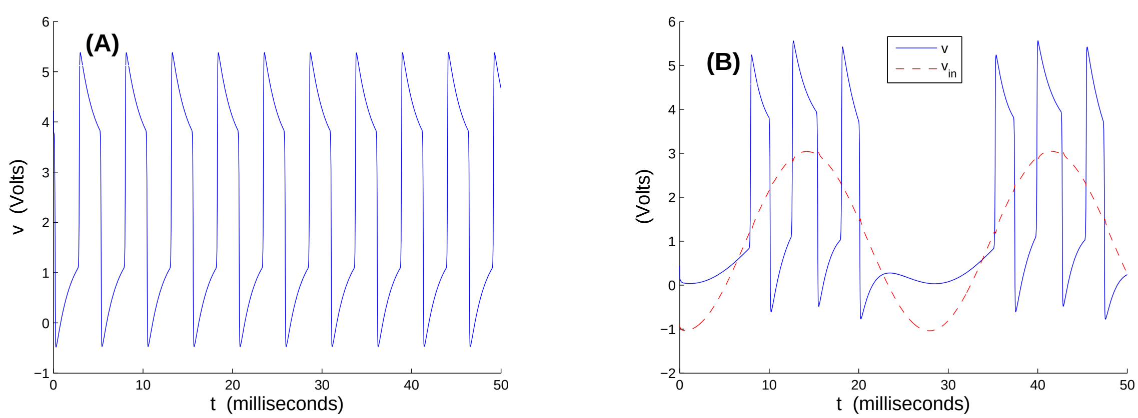
Keener recommends that

$$R_1 = R_2 = 100, \qquad R_3 = 2.4, \quad R_4 = 1 \quad \text{and} \quad R_5 = 10 \quad \text{all } k\Omega$$

and

$$C_1 = 0.01 \quad \text{and} \quad C_2 = 0.5 \quad \text{all } \mu F,$$

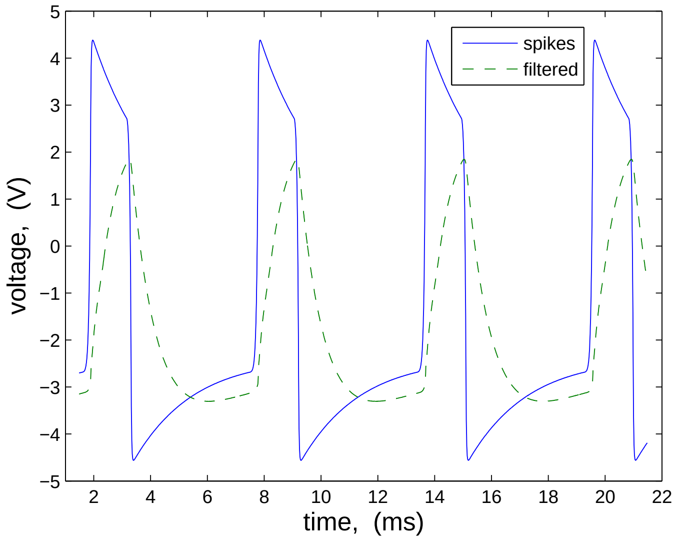
and that op amp 1 be powered by $\pm 15\ V$ and op amp 2 be powered by $\pm 12\ V$. We have followed these recommendations with one exception, we have powered the Sodium op amp with the more convenient ± 9. Our results are presented in Figure 4.3



keendat

Figure 4.3. Expermental testing of the Keener active cell. (A) With $v_{in} = 0$ the circuit generates its own rhythym. (B) With $v_{in} = -1 + 4\sin(2\pi t/36)$ we may modulate this rhythym.

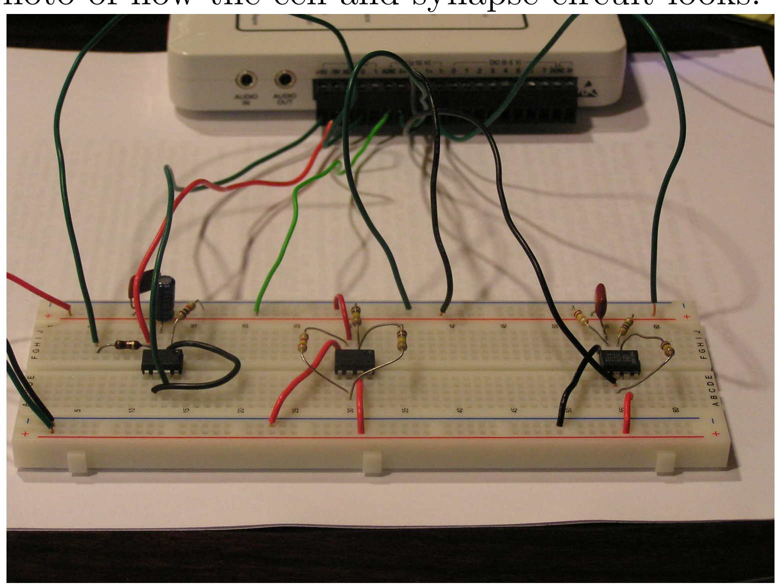
As a final step we run the spikes through a model synapse. That is, a synapse modeled as a lowpass filter.



keenwsyn

Figure 4.4. Spikes, driven with a DC drive (offset) of $-2.8\ V$, and filtered by the first order filter with parameter set (3.15).

and here is a photo of how the cell and synapse circuit looks.



fnsoft **4.2. In Software**