Robust spatial memory maps in flickering neuronal networks: a topological model

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Neurons

Cognition
Cognition

Neurons
Spatial cognition

Neurons
Spatial learning and memory

- navigation
- planning
- imagining spatial scenes
Electrophysiology – place cells
Place field map
– a representation of the cognitive map
Čech’s theorem

P. S. Alexandrov (1929)
E. Čech (1932)
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Place field map

Large-scale topological information

P. S. Alexandrov (1929)
E. Čech (1932)
...Čech’s theorem?

P. S. Alexandrov (1929)
E. Čech (1932)
Temporal relations vs. Spatial relations

Place cell active → Rat runs through a place field
Temporal relations vs. Spatial relations

Another place cell active $\rightarrow$ Rat runs through another place field
Temporal relations vs. Spatial relations

Two place cells coactive \(\rightarrow\) Rat runs through an overlap of two place fields
Spikes $\rightarrow$ Space reconstruction

Temporal simplicial complex

V. de Silva et. al (2007)
Y. Dabaghian et. al (2007)
C. Curto et al. (2008)
Čech’s theorem, spatial overlaps

Rat “discovers” place field overlaps via place cell coactivity

...and learns a spatial map as it builds a Čech’s complex

Y. Dabaghian et. al (2012)
M. Arai et. al (2014)
Place cell coactivity must be detected by the downstream networks.
Reading out topological information

Cell assembly approach:

An active cell assembly elicits a response from a readout neuron

G. Buzsaki, 2010
Reading out topological information

Spiking readout neuron represents a maximal simplex in the Čech complex
Spiking readout neuron represents a maximal simplex in the Čech complex *unreliably*.
Reading out topological information
Reading out topological information

\[ \text{time } t_{-1} \quad \ldots \quad \text{time } t_1 \quad \ldots \quad \text{time } t_2 \quad \ldots \quad \text{time } t_3 \]
Flickering cell assembly network produces a flickering simplicial complex

\[ \text{time } t_{-1} \quad \ldots \quad \text{time } t_1 \quad \ldots \quad \text{time } t_2 \quad \ldots \quad \text{time } t_3 \]
Can a “flickering” complex encode a robust map of space?
...we can test it in a numerical experiment...

Simulate “flickering complex”
a) **Point** – a 0 dimensional (**one 0d**) loop.
b) **Circle** – **one** non-contractible **1d** loop. Note: **one 0d** loop is still (always) there.
c) **Sphere** – all **1D** loops contract into points. So it’s just **one 0D + one 2d** loop.
d) **Torus** – the red loops cannot be shrunk to a point whereas the green loop can.

Total: **two 1d** loops, **one 2d** sheet “looped” on itself, **one 0d** loop

G. Singh et. al., Journal of vision (2008)
Simulations of flickering complex
Flickering complex’s autocorrelations

\[ \langle \langle \cdots \rangle \rangle \]

mean neighbor overlap, \( n = 0.65 \)

asymptotic overlap, \( a = 0.04 \)
Flickering complex’s autocorrelations

mean neighbor overlap, $n = 0.65$

asymptotic overlap, $a = 0.04$
Topological loops in flickering complex

$\ldots t_1 \ldots t_2 \ldots t_3 \ldots t_4 \ldots t_3$
Key parameter: cell assemblies’ mean lifetime
Stability of the loops as a function of simplex’s lifetimes

Number of topological loops

Learning time, minutes

Number of simplexes $\times 10^3$

$W = 2.5$ min
$T_{mn} = 95$ sec
$\zeta = 50\%$
Stability of the loops as a function of simplex’s lifetimes

- Number of topological loops
  - $b_i$
  - $b_2$
  - $b_3$

- Learning time, minutes
  - $W = 4 \text{ min}$
  - $T_{\text{min}} = 117 \text{ sec}$
  - $\alpha = 78\%$

- Number of simplexes $\times 10^3$
  - maximal
  - shared in a timestep
  - 3D
  - 0-3D/10
Stability of the loops as a function of simplex’s lifetimes

- Number of topological loops
- Window shift, seconds

- Learning time, minutes
- Window shift, seconds

- Number of simplexes $\times 10^3$
- Window shift, seconds

- $W = 5$ min
- $T_{min} = 134$ sec

- $\xi = 92\%$
Conclusion: transient networks can encode stable topological maps
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