Optimization Using Surrogate Objectives

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Outline

- The Problem & A One-Shot Solution Approach
- Convergence Theory.
- PDS Results for 51dim Helicopter Example
- The Surrogate Management Framework
- Results for 31 & 11 dim Helicopter Examples
- Results with Different Surrogates
- Conclusions, Status, and Plans

The Problem

$$
\begin{array}{ll}
\text{minimize} & f(x) \\
\text{subject to} & x \in \mathcal{F} \equiv \{x \mid C(x) \leq 0\},
\end{array}
$$

where $f : \Re^n \to \Re \cup \{\infty\}$ may not be continuous, and:

1. $f(x)$ is expensive and has few correct digits.
2. $f(x)$ may fail expensively and unexpectedly for $x \in \mathcal{F}$.
3. $f(x)$ will fail if some specified constraints are violated.

A One-Shot Approach

1. Choose surrogates s_f and s_C based on either:
   (a) simplified physical models; or
   (b) approximations obtained by evaluating f, C at selected design sites, $x_1, \ldots, x_d$, and interpolating or smoothing the function values.
2. Minimize $s_f(x)$ for $s_C(x) \leq 0$ to obtain x_s.
   Every user has their favorite approach for this part.
3. Compute $f(x_s), C(x_s)$ to determine if improvement has been made over the best x found to date.
   What if no improvement was found?

A Basic Filtered Pattern Search Algorithm

Given $x_0 \in M_0$, a conceptual grid on $\Re^n$,
let $X_0 \subset M_0$ be x_0 and the points of M_0 adjacent to x_0.

For $k = 0, 1, \cdots$, do

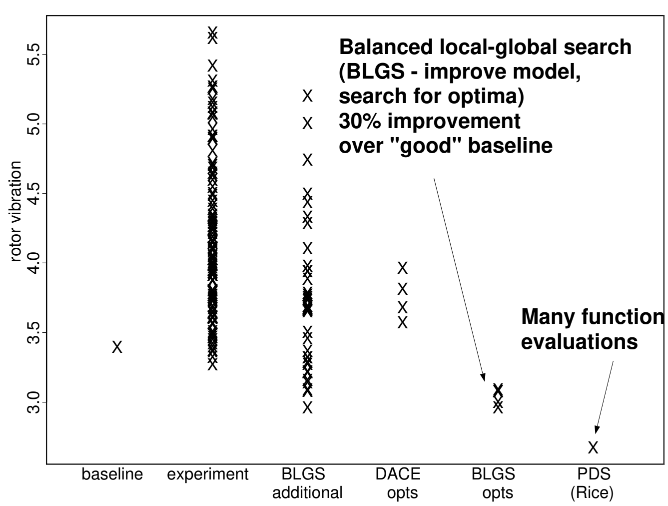
1. Employ some strategy to try to find $x_{k+1} \in M_k$ better than x_k and then set $M_{k+1} = M_k$;

2. Else if x_k is the best point for $x \in X_k$;
   Then set $x_{k+1} = x_k$ and $M_{k+1} = M_k/2$; Else return to 1.

Theorem. $\exists$ limit point x_* at which appropriate first order optimality conditions hold, if $D_S f(x_*)$ and $D_S C(x_*)$ exist.

The search strategy in step 1 is the key to effectiveness.
Step 2 guarantees some minimizer at least.

PDS 51d Helicopter Results



Surrogate Management Framework

Given initial surrogates s_f, s_C and $x_0 \in M_0$, a grid on $\Re^n$, let $X_0 \subset M_0$ be x_0 and the points of M_0 adjacent to x_0.

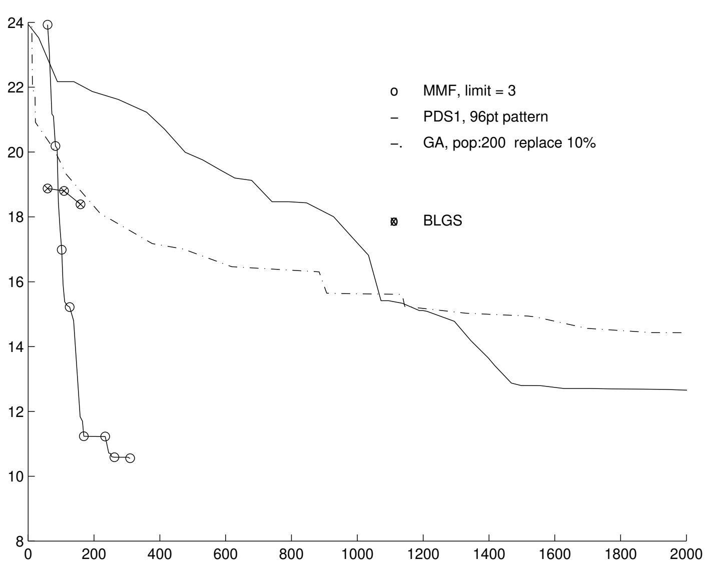
For $k = 0, 1, \cdots$, do

1. Search on s_f, s_C to find $x_{k+1} \in M_k$ better than x_k (wrt f, C) and then set $M_{k+1} = M_k$ and update the surrogates;
2. Else if x_k is the best for $x \in X_k$; Then set $x_{k+1} = x_k$ and $M_{k+1} = M_k/2$ and update the surrogates; Else return to 1.

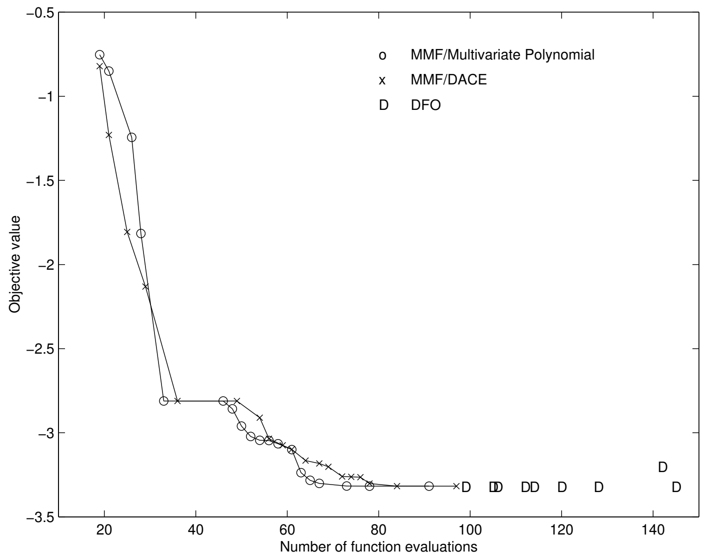
Theorem. $\exists$ limit point x_* at which appropriate first order optimality conditions hold, if $D_S f(x_*)$ and $D_S C(x_*)$ exist.

Theory is independent of goodness of the surrogate.
Performance isn't. **Search step can be a user heuristic**.

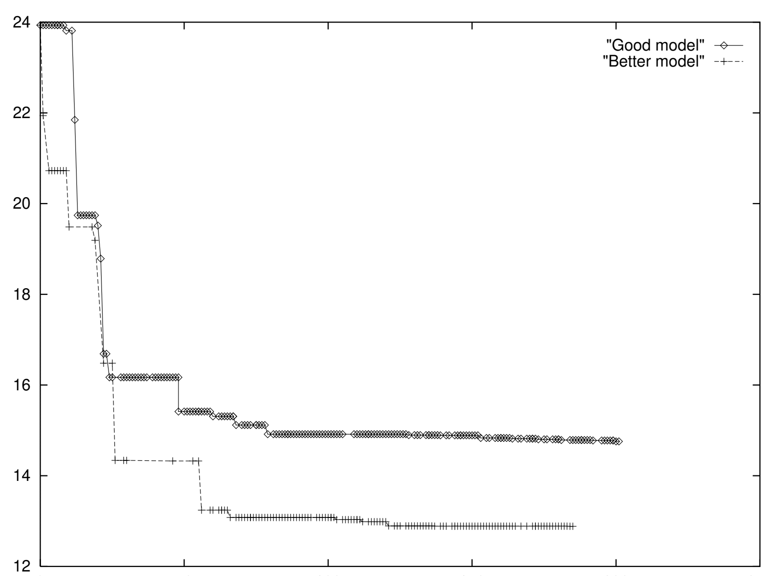
Comparison with Other Methods
- 31d Helicopter Example -



Polynomial vs DACE Surrogates



Recovery from a Bad Surrogate
- 11d Helicopter Example -



Conclusions

- Work is needed on more adaptable surrogates
- Pattern Search can be used directly on $f\&C$, but it may use a lot of function values, especially near the optimum
- SMF needs less accurate surrogates than designers want anyway, and it alleviates some dimensional effects
- SMF seamlessly incorporates user heuristic searches

Status and Plans

- C++ implementation of FPS for general NLP at: www.caam.rice.edu/∼dougm; great success at Boeing.
- MVP - Mixed variable pattern search is analyzed; MatLab code solved design problem from MechE literature. **No reliance on continuous relaxations**.
- Preliminary MatLab code to aid algorithm development for constrained multiobjective is promising.
- Constrained multiobjective product will be interactive surrogate management tools for trade-offs of competing objectives.