HW 1

Due: 31 January 2017 in class

1. If $X \sim N(\mu, \sigma^2)$, compute $E[|X - \mu|]$.

2. Let $X_i \sim N(\mu, \sigma^2)$ for $i = 1\ldots n$. Suppose you estimate $\mu$ and $\sigma$ by

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} X_i, \quad \hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \hat{\mu})^2. \tag{1}$$

   Compute $E[\hat{\sigma}^2]$ and show that it does not equal $\sigma^2$. Modify the formula for $\hat{\sigma}$ so that its expected value is $\sigma^2$.

3. Let $X \sim \text{Exponential}(\lambda)$. Compute the mean $\mu$ and standard deviation $\sigma$ of $X$. Find $P(|X - \mu| \geq k\sigma)$. How does this compare to the bound you get from Chebyshev’s Inequality.

4. Let $X_i \sim \text{Bernoulli}(p)$ for $i = 1\ldots n$. Find a reasonable bound for $P(\frac{1}{n} \sum_{i=1}^{n} X_i < p/2)$ for large $n$ and small $p$.

5. Let $X_i \sim \text{Exponential}(\lambda)$ for $i = 1\ldots n$. Roughly how big will $\max_{i \in [n]} X_i$ be (for arbitrary $\lambda$ and large $n$)? Create and prove a probability bound that shows that it is improbable for the maximum of these random variables to be much larger than your answer.

6. Let $X_i \sim \text{Uniform}([0, L])$ for $i = 1\ldots n$. Roughly how big will $\min_{i \in [n]} X_i$ be (for arbitrary $L > 0$ and large $n$)? Create and prove a probability bound that shows that it is improbable for the minimum of these random variables to be much smaller than your answer.

7. Let $X, Y \sim N(0, 1)$ be independent. Show that $XY$ follows the same distribution as the difference of two independent $\chi^2_1$ random variables. Hint: Expand out $(X + Y)^2 - (X - Y)^2$. 

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