1. Let $Z \sim \mathcal{N}(0, 1)$. Are the following random variables subgaussian? Are they subexponential? Prove your answer.
   
   (a) $Z^2$
   
   (b) $Z^4$
   
   (c) $Z^4 1_{|Z| \leq 4}$ (Here $1_E$ is the indicator of the event $E$)
   
   (d) Geometric distribution with probability $p$

2. Let $A$ be an $N \times n$ real valued matrix. Show that the minimum singular value of $A$ is Lipschitz continuous with Lipschitz constant $1$ as a function of $A$ (with respect to the $\ell^2$ norm if $A$ is considered as an $Nn$ dimensional vector).

3. Let $x_1, x_2, x_3 \sim \mathcal{N}(0, I_n)$ be independent. The goal of this problem is to argue that the triangle formed by these three points is close to being equilateral.
   
   (a) By using a union bound, state and prove a high probability concentration result that the lengths of the three edges are all within a factor of $1.01$ of each other. Your bound should have a probability that approaches $1$ as $n$ approaches $\infty$.
   
   (b) By using a union bound, state and prove a high probability concentration result that each of the triangle’s interior angles are between $0.99 \cdot \pi/3$ and $1.01 \cdot \pi/3$. Your bound should have a probability that approaches $1$ as $n$ approaches $\infty$.

4. (Revised) Let $a_1, \ldots, a_N$ be i.i.d. $\mathcal{N}(0, I_n)$ vectors. Let $\sigma_i \sim \begin{cases} 1 & \text{with prob. } 1/2 \\ -1 & \text{with prob. } 1/2 \end{cases}$ be independent from $a_i$ and each other. Let $A = \sum_{i=1}^N \sigma_i a_i a_i^T$.
   
   (a) Prove that there exists constants $c$, $C$ such that with probability at least $1 - 2e^{-cn}$,
   
   $$\|A\| \leq C(\sqrt{Nn} + n).$$
   
   Here, $\|A\|$ is the spectral norm of $A$.

   (b) Provide a qualitative explanation of the form of this bound on the spectral norm. As part of your answer: why would bound be false without the $\sqrt{Nn}$ term? Why would the bound be false without the $n$ term? Can you provide an intuitive explanation of why the $\sqrt{Nn}$ has the structure it does? That is, if you hadn’t been told this upper bound on the spectral norm of $A$, how could you have reasonably guessed it?