CAAM 336 · DIFFERENTIAL EQUATIONS

Problem Set 1

Distributed Wednesday January 7. Due Wednesday January 16, 2008 in class.

1. [30 points]
   Determine whether each of the following differential equations is an ODE or a PDE, determine its order, and specify whether it is linear or nonlinear. For those that are linear, specify whether they are homogeneous or inhomogeneous.

   (a) $\dfrac{dv}{dx} + \dfrac{2}{x}v = 0$ (b) $\dfrac{\partial v}{\partial t} - 3\dfrac{\partial v}{\partial x} = x - t$

   (c) $\dfrac{\partial u}{\partial t} - \dfrac{\partial}{\partial x}\left[2u\dfrac{\partial u}{\partial x}\right] = 0$ (d) $\dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + \dfrac{\partial^3 u}{\partial x^3} = 0$

   (e) $\dfrac{d^2y}{dx^2} - \mu(1-y^2)\dfrac{dy}{dx} + y = 0$ (f) $\dfrac{d^2}{dx^2}\left[\rho(x)\dfrac{d^2u}{dx^2}\right] = \sin(x)$

2. [30 points]
   Determine whether each of the following functions is a solution of the corresponding differential equation in parts (a), (b), and (c) of question 1, respectively.

   (a) $v(x) = 1/x^2$
   (b) $v(x,t) = t(t+x)$
   (c) $u(x,t) = xe^t$

3. [20 points]
   Is there any *constant* f such that $u(t) = e^t$ is a solution of the ODE

   $$\frac{d^2u}{dt^2} + 4\frac{du}{dt} - 3u = f\,?$$

   If so, specify f. Otherwise, explain why no such f exists.

4. [20 points]
   Suppose that you have a solution u of the equation

   $$a(t)\frac{d^2u}{dt^2} + b(t)\frac{du}{dt} + c(t)u(t) = f(t) \qquad (1)$$

   and that v is a nonzero solution of the homogeneous equation

   $$a(t)\frac{d^2u}{dt^2} + b(t)\frac{du}{dt} + c(t)u(t) = 0.$$

   Explain how to produce infinitely many different solutions of (1).