CAAM 336 · DIFFERENTIAL EQUATIONS

Problem Set 9

Posted Friday 14 March 2008. Due Friday 21 March 2008 in class.

1. [70 points]
   Use the Fourier series method to solve the following initial boundary value problem

$$\begin{aligned} \frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} &= -4x, \qquad 0 \le x \le 1, \qquad t \ge 0 \\ \frac{\partial u}{\partial x}(0,t) &= 1 \\ \frac{\partial u}{\partial x}(1,t) &= 3 \\ u(x,0) &= x^2 \end{aligned}$$

   by doing the following:

   (a) Define a function $p(x)$ so that $v(x,t) = u(x,t) - p(x)$ has homogeneous boundary conditions. Then write down the resulting partial differential equation for v. There are two things to keep in mind here:

      i. While p does not depend on t, it cannot be linear. You must do something else.

      ii. The initial condition and the right hand side for v will be different from those for u.

   (b) Find the Fourier series for the new initial condition $v(x,0)$.

   (c) Use this initial condition to find the Fourier series for $v(x,t)$ for all positive t.

   (d) With the Fourier series for v, write down the Fourier series for u.

   (e) Does this solution stay bounded as you let $t \to \infty$? (From last week's homework, you can answer this question just by looking at the right hand side of the differential equation for v.)

2. [30 points]
   Use the Fourier series method to solve the periodic problem

$$\begin{aligned} \frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} &= 0, \qquad -1 \le x \le 1, \qquad t \ge 0 \\ u(-1,t) &= u(1,t) \\ \frac{\partial u}{\partial x}(-1,t) &= \frac{\partial u}{\partial x}(1,t) \\ u(x,0) &= -x^3 + x \end{aligned}$$