

CAAM 336 · DIFFERENTIAL EQUATIONS

Problem Set 1

Distributed Wednesday January 10. Due Wednesday January 17, 2007 in class.

1. [30 points]

Determine whether each of the following differential equations is an ODE or a PDE, determine its order, and specify whether it is linear or nonlinear. For those that are linear, specify whether they are homogeneous or inhomogeneous.

(a) $\frac{dv}{dx} + \frac{2}{x}v = 0$

(b) $\frac{\partial v}{\partial t} - 3\frac{\partial v}{\partial x} = x - t$

(c) $\frac{\partial u}{\partial t} - \frac{\partial}{\partial x} \left[2u \frac{\partial u}{\partial x} \right] = 0$

(d) $\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0$

(e) $\frac{d^2 y}{dx^2} - \mu(1 - y^2) \frac{dy}{dx} + y = 0$

(f) $\frac{d^2}{dx^2} \left[\rho(x) \frac{d^2 u}{dx^2} \right] = \sin(x)$

2. [30 points]

Determine whether each of the following functions is a solution of the corresponding differential equation in parts (a), (b), and (c) of question 1, respectively.

(a) $v(x) = 1/x^2$

(b) $v(x, t) = t(t + x)$

(c) $u(x, t) = xe^t$

3. [20 points]

Is there any *constant* f such that $u(t) = e^t$ is a solution of the ODE

$$\frac{d^2 u}{dt^2} + 4 \frac{du}{dt} - 3u = f?$$

If so, specify f . Otherwise, explain why no such f exists.

4. [20 points]

Suppose that you have a solution u of the equation

$$a(t) \frac{d^2 u}{dt^2} + b(t) \frac{du}{dt} + c(t)u(t) = f(t) \tag{1}$$

and that v is a nonzero solution of the homogeneous equation

$$a(t) \frac{d^2 u}{dt^2} + b(t) \frac{du}{dt} + c(t)u(t) = 0.$$

Explain how to produce infinitely many different solutions of (1).