Math 126A Spring, 2001

Quiz Three

Show all of your work, and justify your answers. Answers without work or justification will not receive full credit. Leave your answers in exact form; do not try to convert expressions such as \( \pi \) or \( \sqrt{2} \) into decimal approximations. You may not use notes or calculators.

This quiz has two (2) problems. Be sure that you answer all of them.

1. (8 points) Show that the equation

\[ x^2 + y^2 + z^2 + 2x - 4y = 2z \]

represents a sphere, and find its center and radius.
Consider the vectors $\vec{a} = \langle 3, 2, 4 \rangle$ and $\vec{b} = \langle -2, 4, 0 \rangle$.

(a) (3 points) Find $|\vec{a}|$.

(b) (3 points) Find $\vec{a} - 2\vec{b}$.

(c) (3 points) Find a unit vector in the direction of $\vec{a}$.

(d) (3 points) Write $2\vec{a} - 3\vec{b}$ in terms of $\vec{i}$, $\vec{j}$, and $\vec{k}$. 
The equation
\[ x^2 + y^2 + z^2 + 2x - 4y = 2z \]
can be simplified to
\[ (x + 1)^2 + (y - 2)^2 + (z - 1)^2 = 6, \]
which is a sphere centered at \((-1, 2, 1)\) with radius \(\sqrt{6}\). How is this simplification done? First we group terms to get
\[ (x^2 + 2x) + (y^2 - 4y) + (z^2 - 2z) = 0. \]
Now complete the square. To maintain the equality, we must add the terms we add to the left to the right as well:
\[ (x^2 + 2x + 1) + (y^2 - 4y + 4) + (z^2 - 2z + 1) = 1 + 4 + 1. \]
This simplifies to the equation above.

Recall that we are talking about the vectors \(\vec{a} = ⟨3, 2, 4⟩\) and \(\vec{b} = ⟨-2, 4, 0⟩\).

(a) \(|\vec{a}| = \sqrt{(3)^2 + (2)^2 + (4)^2} = \sqrt{29}\)
(b) \(\vec{a} - 2\vec{b} = ⟨3, 2, 4⟩ - 2⟨-2, 4, 0⟩ = ⟨3, 2, 4⟩ - ⟨-4, 8, 0⟩ = ⟨3 - (-4), 2 - 8, 4 - 0⟩ = ⟨7, -6, 4⟩\)
(c) A unit vector in the direction of \(\vec{a}\) is
\[ \frac{\vec{a}}{|\vec{a}|} = \frac{1}{\sqrt{29}} ⟨3, 2, 4⟩ = \langle \frac{3}{\sqrt{29}}, \frac{2}{\sqrt{29}}, \frac{4}{\sqrt{29}} \rangle. \]
(There is one other possible answer, which is \(-1\) times the answer given above.)
(d) \(2\vec{a} - 3\vec{b} = 2⟨3, 2, 4⟩ - 3⟨-2, 4, 0⟩ = ⟨12, -8, 8⟩ = 12\vec{i} - 8\vec{j} + 8\vec{k}\)