MATH 126A SPRING, 2001

Name: ______________					
		Kirk		Kris	
Section	1:30	2:30	1:30	2:30	
(circle one)	AA	AC	AB	AD	

Quiz Four

Show all of your work, and justify your answers. Answers without work or justification will not receive full credit. Leave your answers in *exact* form; do not try to convert expressions such as π or $\sqrt{2}$ into decimal approximations. You may not use notes or calculators.

This quiz has six (6) problems. Be sure that you answer all of them.

We are given three vectors in space

$$\vec{a} = \langle 1, -2, 3 \rangle, \qquad \vec{b} = \langle 0, 4, 5 \rangle, \qquad \vec{c} = \langle -1, 3, 0 \rangle.$$

For each expression below, determine whether or not it makes sense. If it makes sense, evaluate it, being sure to make clear whether it is a vector or a scalar. If it does not make sense, write MAKES NO SENSE.

1 (3 points) $(\vec{a} + \vec{c}) \cdot \vec{b} \ =$

2 (3 points) $\vec{a} \times \vec{b} \times \vec{c} \ =$

3 (3 points) $\vec{a} \cdot (\vec{b} \times \vec{c}) \ =$

Recall that we are given three vectors in space

$$\vec{a} = \langle 1, -2, 3 \rangle, \qquad \vec{b} = \langle 0, 4, 5 \rangle, \qquad \vec{c} = \langle -1, 3, 0 \rangle.$$

4 (3 points) Find a unit vector parallel to $\vec{a}$.

5 (4 points) Find a non-zero vector orthogonal to both $\vec{a}$ and $\vec{b}$.

6 (4 points) Find the scalar projection of $\vec{a}$ in the direction of $\vec{c}$. That is, find the component $\text{comp}_{\vec{c}}\vec{a} = |\vec{a}| \cos(\theta)$ of $\vec{a}$ along $\vec{c}$, where θ is the angle between $\vec{a}$ and $\vec{c}$.

1

$$\begin{aligned}(\vec{a}+\vec{c})\cdot\vec{b} &= (\langle 1,-2,3\rangle + \langle -1,3,0\rangle)\cdot\langle 0,4,5\rangle \\ &= \langle 0,1,3\rangle\cdot\langle 0,4,5\rangle \\ &= (0)(0)+(1)(4)+(3)(5) = 19.\end{aligned}$$

2 $\vec{a}\times\vec{b}\times\vec{c}$ MAKES NO SENSE since $(\vec{a}\times\vec{b})\times\vec{c}$ is not the same as $\vec{a}\times(\vec{b}\times\vec{c})$.

3

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = \langle 1,-2,3\rangle\cdot(\langle 0,4,5\rangle\times\langle -1,3,0\rangle).$$

We compute the cross product as a determinant:

$$\begin{aligned}\langle 0,4,5\rangle\times\langle -1,3,0\rangle &= \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 0 & 4 & 5\\ -1 & 3 & 0\end{vmatrix} \\ &= \vec{i}\begin{vmatrix}4 & 5\\ 3 & 0\end{vmatrix} - \vec{j}\begin{vmatrix}0 & 5\\ -1 & 0\end{vmatrix} + \vec{k}\begin{vmatrix}0 & 4\\ -1 & 3\end{vmatrix} \\ &= \langle -15,-5,4\rangle.\end{aligned}$$

Therefore

$$\begin{aligned}\vec{a}\cdot(\vec{b}\times\vec{c}) &= \langle 1,-2,3\rangle\cdot\langle -15,-5,4\rangle \\ &= (1)(-15)+(-2)(-5)+(3)(4) = 7.\end{aligned}$$

4 A unit vector parallel to $\vec{a}$ is $\frac{\vec{a}}{|\vec{a}|}$. We compute the length of $\vec{a}$:

$$\begin{aligned}|\vec{a}| &= \sqrt{\vec{a}\cdot\vec{a}} \\ &= \sqrt{\langle 1,-2,3\rangle\cdot\langle 1,-2,3\rangle} \\ &= \sqrt{(1)^2+(-2)^2+(3)^2} = \sqrt{14}.\end{aligned}$$

Thus a unit vector parallel to $\vec{a}$ is $\dfrac{1}{\sqrt{14}}\langle 1,-2,3\rangle$ or $\langle\dfrac{1}{\sqrt{14}},-\dfrac{2}{\sqrt{14}},\dfrac{3}{\sqrt{14}}\rangle$

5 A non-zero vector orthogonal to both $\vec{a}$ and $\vec{b}$ is simply $\vec{a}\times\vec{b}$. We compute this:

$$\begin{aligned}\langle 1,-2,3\rangle\times\langle 0,4,5\rangle &= \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & -2 & 3\\ 0 & 4 & 5\end{vmatrix} \\ &= \vec{i}\begin{vmatrix}-2 & 3\\ 4 & 5\end{vmatrix} - \vec{j}\begin{vmatrix}1 & 3\\ 0 & 5\end{vmatrix} + \vec{k}\begin{vmatrix}1 & -2\\ 0 & 4\end{vmatrix} \\ &= \langle -22,-5,4\rangle.\end{aligned}$$

6 The scalar projection of $\vec{a}$ in the direction of $\vec{c}$ is $\text{comp}_{\vec{c}}\vec{a} = |\vec{a}|\cos(\theta) = \frac{\vec{a}\cdot\vec{c}}{|\vec{c}|}$. This is just another computation:

$$\begin{aligned}
\text{comp}_{\vec{c}}\vec{a} &= \frac{\vec{a}\cdot\vec{c}}{|\vec{c}|} \\
&= \frac{\langle 1,-2,3\rangle \cdot \langle -1,3,0\rangle}{|\langle -1,3,0\rangle|} \\
&= \frac{(1)(-1)+(-2)(3)+(3)(0)}{\sqrt{(-1)^2+3^2+0^2}} \\
&= \frac{-7}{\sqrt{10}}.
\end{aligned}$$