Quiz Five

Show all of your work, and justify your answers. Answers without work or justification will not receive full credit. Leave your answers in exact form; do not try to convert expressions such as \( \pi \) or \( \sqrt{2} \) into decimal approximations. You may not use notes or calculators.

This quiz has three (3) problems. Be sure that you answer all of them.

1. (8 points) Find the area of the surface obtained by rotating the curve \( x = 2 \cos^3 \theta, y = 2 \sin^3 \theta, \) \( 0 \leq \theta \leq \pi/2, \) about the \( x \)-axis.

2. (6 points) Find the unit tangent vector \( T(t) \) for the curve \( r(t) = (t, 1 - t, 3t^2) \). Your answer should be a function of \( t \).
3. (3 points each) Match the parametric or vector equations with the graphs (labeled I – IV). Notice that there are more graphs than equations.

(a) \( x = \cos t, \quad y = \sin t, \quad z = \sin 3t \) matches graph [ ]

(b) \( \mathbf{r}(t) = \langle t, t^2, t^4 \rangle \) matches graph [ ]

I. [ ]

II. [ ]

III. [ ]

IV. [ ]
1. The area of the surface obtained by rotating the curve \( x = 2 \cos 3\theta, y = 2 \sin 3\theta, 0 \leq \theta \leq \pi/2, \) about the \( x \)-axis is given by the formula

\[
A = \int_{t=a}^{t=b} y \, ds = \int_{t=a}^{t=b} y \sqrt{(dx/dt)^2 + (dy/dt)^2} \, dt
\]

In terms of our functions (and \( \theta \), not \( t \)), this is

\[
A = \int_{\theta=0}^{\theta=\pi/2} y \, ds = \int_{\theta=0}^{\theta=\pi/2} 2 \sin^3 \theta \sqrt{(3 \sin^2 \theta \cos \theta)^2 + (-3 \cos^2 \theta \sin \theta)^2} \, d\theta.
\]

Simplifying this, we get

\[
A = 6 \int_{0}^{\pi/2} \sin^3 \theta \sqrt{\sin^2 \theta \cos^2 \theta (\sin^2 \theta + \cos^2 \theta)} \, d\theta
\]

\[
= 6 \int_{0}^{\pi/2} \sin^4 \theta \cos \theta \, d\theta
\]

\[
= \frac{6}{5} \sin^5 \theta \bigg|_0^{\pi/2}
\]

\[
= \frac{6}{5}.
\]

Thus the surface area is \( 6/5 \) square units.

2. The unit tangent vector \( \mathbf{T}(t) \) for the curve \( \mathbf{r}(t) = \langle t, 1 - t, 3t^2 \rangle \) is given by the formula

\[
\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\left| \mathbf{r}'(t) \right|}.
\]

Hence, since \( \mathbf{r}'(t) = \langle 1, -1, 6t \rangle \) and \( \left| \mathbf{r}'(t) \right| = \sqrt{36t^2 + 2} \), we have

\[
\mathbf{T}(t) = \frac{1}{\sqrt{36t^2 + 2}} \langle 1, -1, 6t \rangle.
\]

3. (3 points each)

4. (3 points each)

   (a) The curve \( x = \cos t, \quad y = \sin t, \quad z = \sin 3t \) matches graph II:

   (b) \( \mathbf{r}(t) = \langle t, t^2, t^4 \rangle \) matches graph III: