

Name: _____

Quiz Six

	Kirk	Kris
Section	1:30	2:30
(circle one)	AA	AC
	AB	AD

Show all of your work, and justify your answers. Answers without work or justification will not receive full credit. Leave your answers in *exact* form; do not try to convert expressions such as π or $\sqrt{2}$ into decimal approximations. You may not use notes or calculators.

This quiz has three (3) problems. Be sure that you answer all of them.

You may (or may not) find the following formulas useful for this quiz.

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} \quad \mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} \quad \mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$$

$$\kappa = \left| \frac{d\mathbf{T}}{ds} \right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

(6 points) Find the curvature of the curve $\mathbf{r}(t) = \langle t^3, t^2, t \rangle$ at the point $(-1, 1, -1)$ ($t = -1$).

(6 points) For the same curve, $\mathbf{r}(t) = \langle t^3, t^2, t \rangle$, find the equation of the normal plane at the point $(-1, 1, -1)$ ($t = -1$).

3 A projectile is launched from the origin with an angle of elevation of 60° and an initial speed 200 m/s.

(a) (4 points) Find equations for $\mathbf{r}(t)$, the position of the projectile at time t .

(b) (4 points) What is the height of the projectile when it is 200 meters down range? (Use 9.8 m/s^2 as an approximation for g .)

- 1 To compute the curvature of the curve $\mathbf{r}(t) = \langle t^3, t^2, t \rangle$ at the point $(-1, 1, -1)$ ($t = -1$), we will use the formula $\kappa = \frac{|\mathbf{r}'(-1) \times \mathbf{r}''(-1)|}{|\mathbf{r}'(-1)|^3}$. We compute: $\mathbf{r}'(t) = \langle 3t^2, 2t, 1 \rangle$, $\mathbf{r}''(t) = \langle 6t, 2, 0 \rangle$, and so $\mathbf{r}'(-1) = \langle 3, -2, 1 \rangle$ and $\mathbf{r}''(-1) = \langle -6, 2, 0 \rangle$. Thus $|\mathbf{r}'(-1)|^3 = (3^2 + (-2)^2 + 1^2)^{3/2} = 14\sqrt{14}$ and

$$\mathbf{r}'(-1) \times \mathbf{r}''(-1) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 1 \\ -6 & 2 & 0 \end{vmatrix} = \langle -2, 6, -6 \rangle,$$

so $|\mathbf{r}'(-1) \times \mathbf{r}''(-1)| = |\langle -2, 6, -6 \rangle| = 2\sqrt{19}$. Hence $\kappa = \frac{2\sqrt{19}}{14\sqrt{14}} = \frac{\sqrt{19}}{7\sqrt{14}}$.

- 2 The normal of the normal plane to $\mathbf{r}(t) = \langle t^3, t^2, t \rangle$, at the point $(-1, 1, -1)$ ($t = -1$) is simply the tangent vector $\mathbf{r}'(-1)$. Thus $\mathbf{n} = \mathbf{r}'(-1) = \langle 3, -2, 1 \rangle$ at the point $(-1, 1, -1)$. The plane through this point with this normal vector is simply $3(x + 1) - 2(y - 1) + 1(z + 1) = 0$ or $3x - 2y + z = -6$.

- 3 (a) To find equations for $\mathbf{r}(t)$, the position of the projectile at time t , we first note that $\mathbf{a}(t) = \langle 0, -g \rangle$, $\mathbf{v}(0) = \langle v_0 \cos(\alpha), v_0 \sin(\alpha) \rangle = \langle 200 \cos(60^\circ), 200 \sin(60^\circ) \rangle = \langle 100, 100\sqrt{3} \rangle$ (this is the initial velocity), and $\mathbf{r}(0) = \langle 0, 0 \rangle$ (we start at the origin). Now we integrate twice because $\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t)$ and $\mathbf{v}(t) = \mathbf{r}'(t)$. Thus $\mathbf{r}'(t) = \langle 0, -gt \rangle + \mathbf{v}(0) = \langle 100, 100\sqrt{3} - gt \rangle$ and $\mathbf{r}(t) = \langle 100t, 100\sqrt{3}t - \frac{g}{2}t^2 \rangle + \mathbf{r}(0) = \langle 100t, 100\sqrt{3}t - \frac{g}{2}t^2 \rangle$.
- (b) From part (a), we have $x(t) = 100t$ and $y(t) = 100\sqrt{3}t - \frac{g}{2}t^2$. We are asked what $y(t)$ is when $x(t) = 200$. This is when $t = 2$, so the height is $y(2) = 100\sqrt{3}(2) - \frac{g}{2}(2)^2 = 200\sqrt{3} - 19.6$ meters. (This is roughly 326.8 meters.)