

Your Name

Your Signature

Student ID #

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Problem	Total Points	Score
1	14	
2	17	
3	8	
4	10	
5	12	
6	13	
7	8	
8	18	
Total	100	

- This exam is closed book. You may use one  $8\frac{1}{2} \times 11$  sheet of notes.
- Do not share notes.
- Calculators are not allowed.
- In order to receive credit, you must show your work. You must also justify all conclusions you make. Do not assume something is obvious. If you feel something is clear enough to not necessitate algebra, write a sentence or two explaining your reasoning. Do not do computations in your head. Instead, write them out on the exam paper.
- Place a box around **YOUR FINAL ANSWER** to each question.
- If you are unable to simplify an answer, leave it in a form that can be put in to a calculator. Do not assume that every answer will be simple.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

1 (14 points) Find all  $p$  such that  $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^p}$  converges absolutely. Find all  $p$  such that it converges conditionally.

2 (17 points)

(a) (8 points) Find a MacLaurin series for  $f(x) = \int_0^x \frac{\tan^{-1}t}{t} dt$ .

(b) (4 points) Find the interval of convergence of the MacLaurin series for  $f(x)$ .

(c) (5 points) Find  $f^{(33)}(0)$ .

- 3 (8 points) Consider a box with mass 4kg which is on a ramp which is at an angle of  $30^\circ$  with the ground. It's a really icy ramp, and the box starts sliding (no friction). How fast is the box moving after two seconds?

4 (10 points) Consider the planes

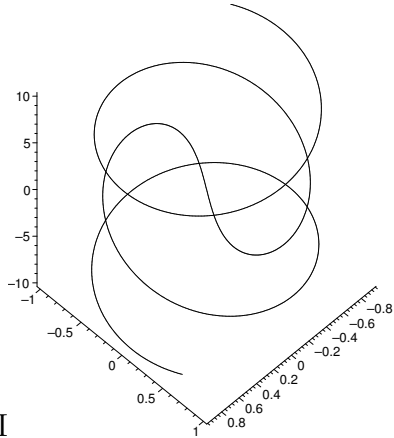
$$x - y - 2z + 4 = 0$$

$$3x + y - 2z = 0$$

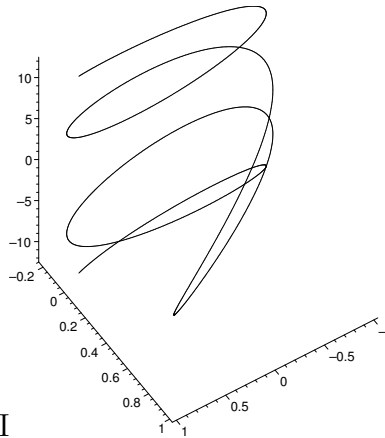
(a) (4 points) Find the angle between these two planes.

(b) (6 points) Find the line contained in both of these planes.

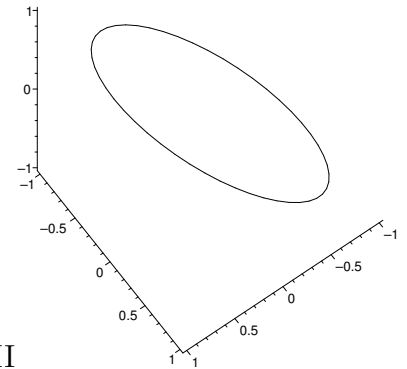
5 (12 points)



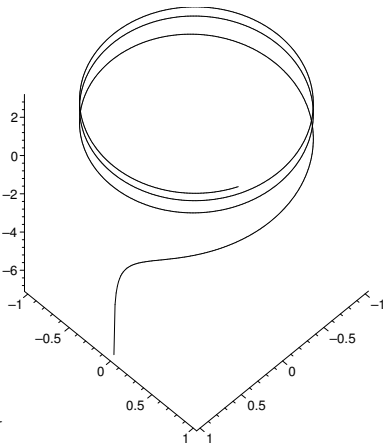
I



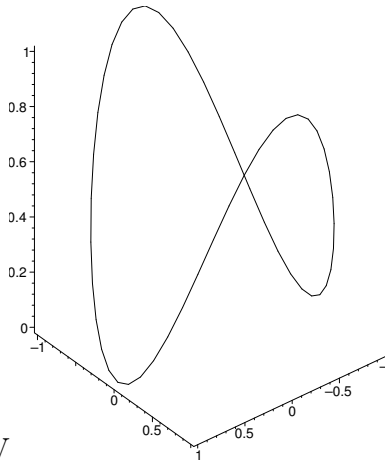
II



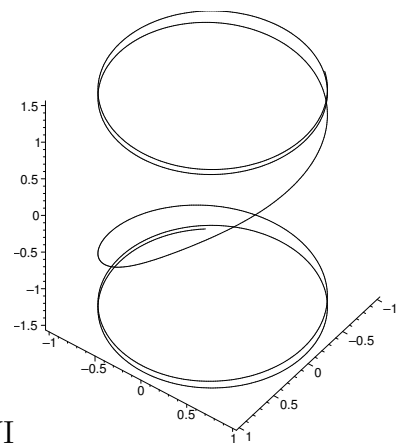
III



IV



V



VI

Each of these pictures corresponds to one of the following functions:

- (a)  $\vec{r}(t) = \langle \cos t, \sin t, \cos t \rangle$
- (b)  $\vec{r}(t) = \langle \cos t, \sin t, \sin^2 t \rangle$
- (c)  $\vec{r}(t) = \langle \cos t, \sin t, \tan^{-1} t \rangle$
- (d)  $\vec{r}(t) = \langle \frac{2}{\pi} \tan^{-1} t \cos t, \sin t, t \rangle$
- (e)  $\vec{r}(t) = \langle \cos t, \frac{\sin t}{t}, t \rangle$
- (f)  $\vec{r}(t) = \langle \cos t, \sin t, \ln t \rangle$

Tell which picture matches which function, and justify your conclusions.

Feel free to use this page if there is not enough room on the previous page to give full explanations.

6 (13 points) Consider the function

$$u(x, t) = \ln(x + t) + \ln(x - t)$$

- (a) (5 points) What is the domain and range of this function? Feel free to draw the domain in the  $t$ - $x$  plane ( $t$  horizontal,  $x$  vertical).

- (b) (5 points) A function is said to satisfy the *wave equation* if  $u_{tt} = c^2 u_{xx}$ . Show that if  $c=1$ , then  $u(x, t)$ , given above, satisfies the wave equation on its domain.

- (c) (3 points) Find the equation of the tangent plane to  $u(x, t)$  at the point  $(x_0, t_0) = (\frac{3}{2}, \frac{1}{2})$ .

- 7 (8 points) Show that the curvature at any point on any line is zero.

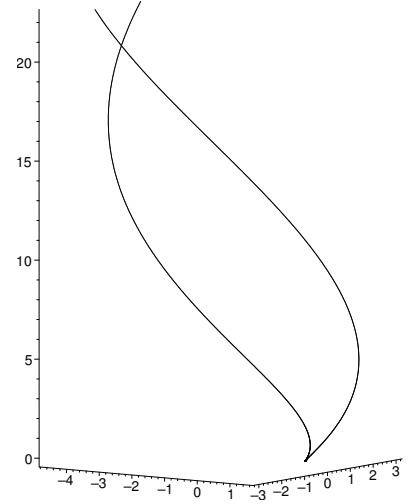
8 (18 points) Consider the following parametric equation:

$$x(t) = \cos t + t \sin t$$

$$y(t) = \sin t - t \cos t$$

$$z(t) = t^2$$

- (a) (8 points) Find the tangent, normal, and binormal vectors to this curve for all positive  $t$ .



Here are those same parametric equations again.

$$x(t) = \cos t + t \sin t$$

$$y(t) = \sin t - t \cos t$$

$$z(t) = t^2$$

(b) (4 points) Find the arclength of the curve from  $t = \frac{\pi}{2}$  to  $t = \frac{3\pi}{2}$ .

(c) (3 points) Observe on the graph that there is a cusp. When does this happen (what value of  $t$ ) and where does this happen (in space)?

(d) (3 points) What is the equation of the normal plane to this curve at time  $t = \frac{\pi}{2}$ ?