

Your Name

Your Signature

Student ID #

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| Problem | Total Points | Score |
|---------|--------------|-------|
| 1       | 14           |       |
| 2       | 14           |       |
| 3       | 14           |       |
| 4       | 10           |       |
| 5       | 12           |       |
| 6       | 14           |       |
| 7       | 22           |       |
| Total   | 100          |       |

- This exam is closed book. You may use one  $8\frac{1}{2} \times 11$  sheet of notes.
- Do not share notes.
- Calculators are not allowed.
- In order to receive credit, you must show your work. You must also justify all conclusions you make. Do not assume something is obvious. If you feel something is clear enough to not necessitate algebra, write a sentence or two explaining your reasoning. Do not do computations in your head. Instead, write them out on the exam paper.
- Place a box around **YOUR FINAL ANSWER** to each question.
- If you are unable to simplify an answer, leave it in a form that can be put in to a calculator. Do not assume that every answer will be simple.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

1 (14 points) Test the following series for absolute convergence, conditional convergence, or divergence.

(a) (7 points) 
$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$$

(b) (7 points) 
$$\sum_{n=1}^{\infty} (-1)^n [\ln(n+1) - \ln(n)]$$

2 (14 points)

(a) (10 points) Find a MacLaurin series for  $f(x) = \int_0^x \frac{e^{t^2} - 1}{t} dt$ .

(b) (4 points) What is  $f^{(40)}(0)$ ?

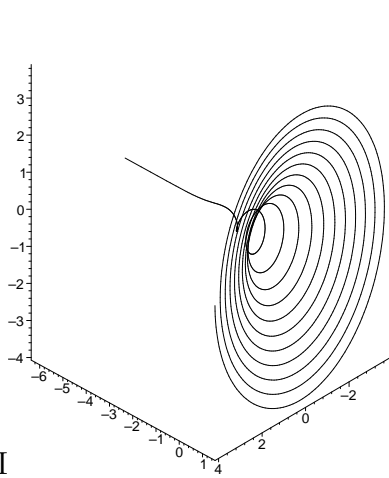
3 (14 points) Consider the lines

$$\begin{aligned}\mathbf{r}_1(t) &= \langle -1 + 5t, 1 + 2t, t \rangle \\ \mathbf{r}_2(t) &= \langle 1 + 3t, 1 + 2t, 2 - t \rangle \\ \mathbf{r}_3(t) &= \langle 4t, 2 + t, -1 + 2t \rangle\end{aligned}$$

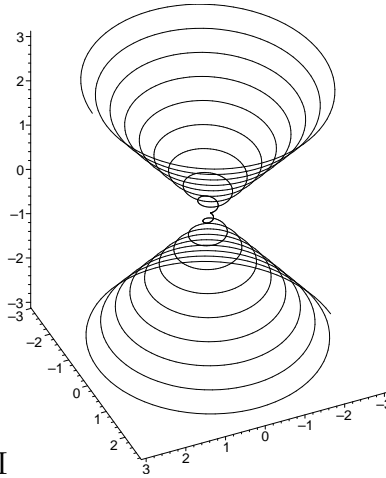
1. (4 points) These three lines intersect in a single point. Find that point. Make sure to confirm that it lies on *all* three lines.
2. (5 points) Show that these three lines are coplanar.
3. (5 points) Now you know that these three lines lie in a single plane. Find that plane.

- 4 (10 points) Show that the curvature on any point on a circle of radius  $r$  is  $\frac{1}{r}$ . If you like, you can assume that the circle lies in the  $xy$ -plane and is centered at the origin (i.e. has equation  $x^2 + y^2 = r^2$ ). Also, you can parameterize this circle if you prefer to compute curvature that way, but you don't have to do that.

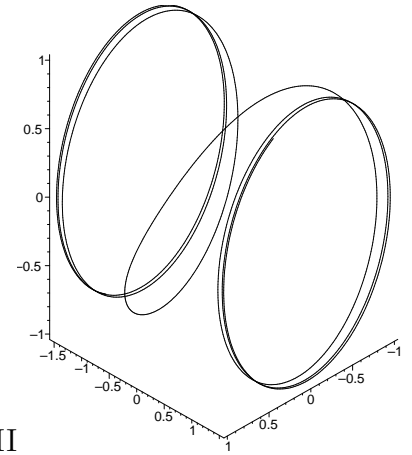
5 (12 points)



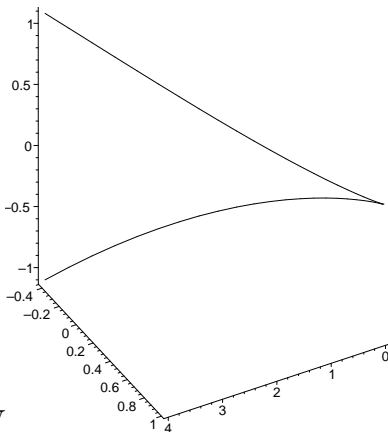
I



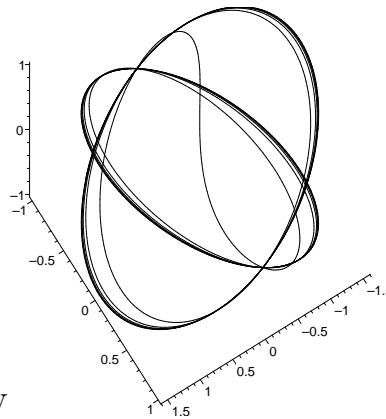
II



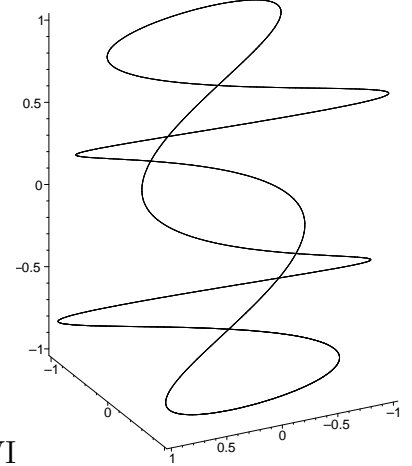
III



IV



V



VI

Each of these pictures corresponds to one of the following functions:

- |   |  |
|---|--|
| (a) $\mathbf{r}(t) = \langle \cos(5t), \sin(3t), \cos t \rangle$            | (b) $\mathbf{r}(t) = \langle t^2, \cos t, \sin t - t \rangle$            |
| (c) $\mathbf{r}(t) = \langle t \cos(6\pi t), t \sin(6\pi t), t \rangle$     | (d) $\mathbf{r}(t) = \langle \tan^{-1} t \cos t, \sin t, \cos t \rangle$ |
| (e) $\mathbf{r}(t) = \langle t \cos(6\pi t), \ln t, t \sin(6\pi t) \rangle$ | (f) $\mathbf{r}(t) = \langle \cos t, \tan^{-1} t, \sin t \rangle$        |

Tell which picture matches which function, and justify your conclusions.

Feel free to use this page if there is not enough room on the previous page to give full explanations.

6 (14 points) Consider the function

$$u(x, y) = \tan^{-1} \frac{y}{x}$$

(a) (3 points) What is the domain and range of this function?

(b) (7 points) A function is called harmonic if  $\nabla^2 u = 0$ ; where  $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ . Show that  $u(x, y)$  is a harmonic function on its domain.

(c) (4 points) Find the equation of the tangent plane to  $u(x, y)$  at the point  $(1, 1, \frac{\pi}{4})$ .

7 (22 points)  
equations:

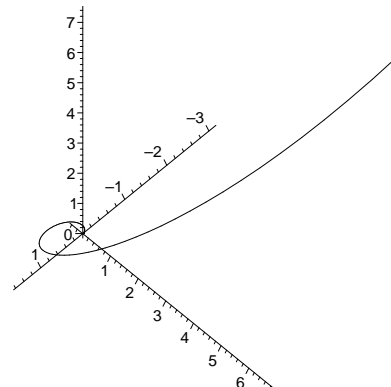
Consider the following parametric

$$x(t) = e^t \cos t$$

$$y(t) = e^t \sin t$$

$$z(t) = e^t$$

(a) (14 points) Find  $\mathbf{T}$ ,  $\mathbf{N}$ , and  $\mathbf{B}$  for this curve.



(b) (4 points) Find the arclength of this curve between the points  $(0, 0, 0)$  and  $(1, 0, 1)$ .

(c) (4 points) Find the tangent line to this curve at  $(1, 0, 1)$ .