

Quiz One

Show all of your work, and justify your answers. If you do not, you will lose points. You may not use notes or calculators.

This quiz has two (2) problems, and the second depends on your answers to the first. Please read the entire quiz before starting. Make sure to answer both questions.

1 For the following three sequences, determine if $\lim_{n \rightarrow \infty} a_n$ exists. If it does, find the limit.

(a) $a_n = \frac{1}{2 + 3^{-n}}$

For this sequence, note that $3^{-n} \rightarrow 0$ as $n \rightarrow \infty$, so $a_n \rightarrow \frac{1}{2+0} = \frac{1}{2}$ as $n \rightarrow \infty$.

(b) $a_n = \cos\left(\frac{n\pi}{2}\right)$

As for this sequence, a_n oscillates between the numbers -1, 0, and 1. The important part is that it doesn't settle to any number, so it cannot converge.

(c) $a_n = 2^{-2n}3^n$

$2^{-2n}3^n = \frac{3^n}{2^{2n}} = \frac{3^n}{4^n} = \left(\frac{3}{4}\right)^n$, so since $\frac{3}{4} < 1$, $a_n \rightarrow 0$.

2 For each of the three sequences in part (a), does the series $\sum_{n=1}^{\infty} a_n$ converge or diverge? Justify your answer, and make sure you check all three sequences.

(d) This sequence converges, but it converges to $\frac{1}{2}$, which isn't zero, so the corresponding series cannot converge.

(e) The sequence diverges, so the series must also diverge. Or, if you look at the terms of the series, s_n will be either -1 or 0, so the series diverges.

(f) The series here is a geometric series, with $r < 1$, so the series converges.