

**Quiz Four**

Show all of your work. You may not use notes or calculators.

- 1 Consider the following two vectors:  $\mathbf{a} = \langle 3, 0, 3 \rangle$ ,  $\mathbf{b} = \langle 4, -1, -1 \rangle$ . For this problem, do *one* of the following two things:

- (a) Find the scalar projection of  $\mathbf{a}$  onto  $\mathbf{b}$ .  
 (b) Find the angle between  $\mathbf{a}$  and  $\mathbf{b}$ .

For the first part, it is a matter of remembering the scalar projection formula.

$$\text{comp}_{\mathbf{b}}\mathbf{a} = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{b}\|} = \frac{\langle 3, 0, 3 \rangle \cdot \langle 4, -1, -1 \rangle}{\|\langle 4, -1, -1 \rangle\|} = \frac{12 + 0 - 3}{\sqrt{16 + 1 + 1}} = \frac{9}{\sqrt{18}} = \frac{3}{\sqrt{2}}$$

For the second part, this all comes down to the formula  $\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\|\cos\theta$ .

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|\|\mathbf{b}\|} = \frac{\langle 3, 0, 3 \rangle \cdot \langle 4, -1, -1 \rangle}{\sqrt{3^2 + 0 + 3^2}\sqrt{4^2 + 1 + 1}} = \frac{12 + 0 - 3}{\sqrt{18}\sqrt{18}} = \frac{9}{18} = \frac{1}{2}$$

So  $\cos\theta = \frac{1}{2}$ , and  $\theta = \frac{\pi}{3}$ .

- 2 Find the center and radius of the following sphere:

$$x^2 - 2x + y^2 + z^2 + 6z + 6 = 0$$

This is all about completing the square.

$$x^2 - 2x + 1 + y^2 + z^2 + 6z + 6 - 1 = 0$$

$$(x - 1)^2 + y^2 + z^2 + 6z + 5 = 0$$

$$(x - 1)^2 + y^2 + z^2 + 6z + 9 + 5 - 9 = 0$$

$$(x - 1)^2 + y^2 + (z + 3)^2 - 4 = 0$$

$$(x - 1)^2 + y^2 + (z + 3)^2 = 4$$

So the center is  $(1, 0, -3)$  and the radius is 2.