• This exam is closed book, and no notes are allowed.

• Calculators are not allowed.

• In order to receive credit, you must show your work. Do not do computations in your head. Instead, write them out on the exam paper.

• Place a box around \textbf{YOUR FINAL ANSWER} to each question.

• If you need more room, use the backs of the pages and indicate to me that you have done so.

• Raise your hand if you have a question.

• A table of Laplace transforms has been included with the test. Feel free to separate it from the rest of the exam and use it as a reference.

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<th>Problem</th>
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(15 points) Consider the following first order differential equation:

\[(t^2 + 2)y' + 2ty = 0.\]

There are three different ways to solve this differential equation. Solve it all three ways, labelling each method (5 points each method).

(a) (5 points) Method 1:

(b) (5 points) Method 2:
$(t^2 + 2)y' + 2ty = 0.$

(c) (5 points) Method 3:
(a) (8 points) Find the general solution of the following differential equation:

\[ x^2y'' + 3xy' + y = 0. \]

(b) (3 points) What is the behavior of \( y(x) \) as \( x \to 0 \)? As \( x \to \infty \)?
(a) (6 points) There is a 100 gallon tank of pure water. Flowing into it at one gallon per minute is water containing two kilograms of salt per gallon of water. Water is flowing out of the tank at the rate of three gallons per minute. Find the amount of salt in the tank at any time $t$ that there is still water in the tank.

(b) (4 points) Find the concentration of salt in the tank at the exact moment the water runs out of the tank.
(8 points) Find the general solution of the following differential equation:

\[ 2y^{iv} - 2y''' - y'' + y = 0. \]
(11 points) Consider the following differential equation:

\[ y'' - 5y' - 6y = 10e^t - 7e^{-t}, \quad y(0) = 0, \text{ and } y'(0) = 0. \]

Find the solution of this initial value problem using the method of undetermined coefficients.
For the same differential equation as in problem 6,

\[ y'' - 5y' - 6y = 10e^t - 7e^{-t}, \text{ } y(0) = 0, \text{ and } y'(0) = 0, \]

find the solution using the Laplace transform.
To the right is the equation of $f(t)$. Find the Laplace transform of $f(t)$. You will need to use the fact that

$$\sin \theta = \sin(\theta + 2\pi)$$
Find the solution of the following differential equation:

\[ y'''' + y''' - 6y' = \delta(t - 1), \quad y(0) = 0, \quad y'(0) = 1, \quad \text{and} \quad y''(0) = -1 \]