

1. Put  $\sin(3t) + \cos(3t)$  in the form  $A \sin(Bt + C)$ . Recall that  $\sin(\theta + \phi) = \sin(\theta) \cos(\phi) + \cos(\theta) \sin(\phi)$ .

This is the same sort of thing as the first proficiency test and the practice test. Expanding the formula  $A \sin(Bt + C)$ , you get  $A \sin(Bt) \cos(C) + A \cos(Bt) \sin(C)$ . So  $B = 3$ , and equating the  $\sin(3t)$  terms and the  $\cos(3t)$  terms, respectively, you get:

$$1 = A \cos(C) \tag{1}$$

$$1 = A \sin(C). \tag{2}$$

Divide (2) by (1) to get

$$1 = \tan(C)$$

This means that  $C = \frac{\pi}{4}$ . Plugging this back in to either equation (1) or (2), you get that  $A = \sqrt{2}$ . So  $\sin(3t) + \cos(3t) = \sqrt{2} \sin(3t + \frac{\pi}{4})$ .

2. Solve the following expression for  $y$  and simplify:

$$\ln\left(\frac{y}{4}\right) = x^2 - \ln(x^2 + x) + \ln(x), \text{ where } x > 0 \text{ and } y > 0.$$

This was the problem most missed by everyone. The key bits that messed most everyone up is that  $e^{a+b} = e^a e^b$  and  $e^{-a} = \frac{1}{e^a}$ . It is VERY important to understand how this stuff works. Get this information down before the first test.

Anyway, on to the problem itself. There are a number of ways to start this. I will approach it the way most people tried to do it. So, exponentiate both sides:

$$\begin{aligned} e^{\ln(\frac{y}{4})} &= e^{[x^2 - \ln(x^2+x) + \ln(x)]} \\ \frac{y}{4} &= e^{x^2} e^{-\ln(x^2+x)} e^{\ln(x)} \\ \frac{y}{4} &= \frac{e^{x^2} e^{\ln(x)}}{e^{\ln(x^2+x)}} \\ \frac{y}{4} &= \frac{e^{x^2} x}{x^2 + x} \\ \frac{y}{4} &= \frac{e^{x^2}}{x + 1} \\ y &= \frac{4e^{x^2}}{x + 1}. \end{aligned}$$

3.  $\int \frac{x^3}{x^2 - 4} dx =$

The first thing you need to remember here is that if you have a rational function (that is, a polynomial divided by a polynomial), and the order of the top is greater than or equal to the order of the bottom, you MUST divide before doing partial fractions. Well, actually, in this situation, you can make a  $u$  substitution, letting  $u = x^2 - 4$  and everything works out wonderfully, but that's really just a lucky coincidence. Hmmmm...I'm having trouble figuring out how to illustrate long division in LaTeX. Maybe later. Here's what you get when you do the long division:

$$\int x + \frac{4x}{x^2 - 4} dx$$

Then at this point you can either do partial fractions, getting that

$$\frac{4x}{x^2 - 4} = \frac{2}{x - 2} + \frac{2}{x + 2}$$

This gives you the integral

$$\int x + \frac{2}{x - 2} + \frac{2}{x + 2} dx = \frac{1}{2}x^2 + 2 \ln |x - 2| + 2 \ln |x + 2| + C$$

or you can recognize that if you just pull off the two in the numerator of the fraction, you actually have the form of  $\frac{u'}{u}$ , and you get

$$\int x + 2 \frac{2x}{x^2 - 4} dx = \frac{1}{2}x^2 + 2 \ln |x^2 - 4| + C.$$

Either way is just as good.