Math 324E **Final Exam** **Autumn 2005**

Your Name

Your Signature

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
Total	50	

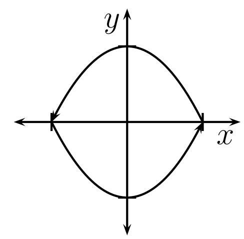
- This exam is closed book. There is a page of formulas attached to the back of the test. Feel free to tear it off to use as reference on the test. However, if you use it for work, you should probably hand it in with your test.
- Only non-graphing scientific calculators are allowed.
- In order to receive credit, you must show your work. You must also justify all conclusions you make. Do not assume something is obvious. If you feel something is clear enough to not necessitate algebra, write a sentence or two explaining your reasoning. Do not do computations in your head. Instead, write them out on the exam paper.
- Place a box around YOUR FINAL ANSWER to each question.
- If you are unable to simplify an anwer, leave it in a form that can be put in to a calculator. Do not assume that every answer will be simple.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

[1] (10 points)

Evaluate

$$\int_C -xy\,dx + \frac{1}{2}x^2\,dy,$$

where C is the curve given by $\mathbf{r}(t) = \langle t, t^2 - 1\rangle$ for $-1 \le t \le 1$ and $\mathbf{r}(t) = \langle 2 - t, -t^2 + 4t - 3\rangle$ for $1 \le t \le 3$. See the figure to the right.

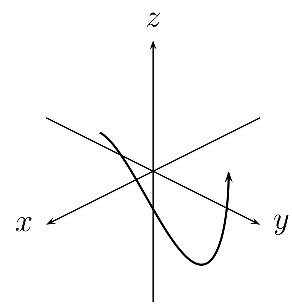


2 (10 points)

Evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r},$$

where $\mathbf{F}(x, y, z) = \langle -yz, xz, x^2 \rangle$ and C is the curve defined by $\mathbf{r}(\theta) = \langle \cos\theta, \sin\theta, \cos(2\theta) \rangle$ where $0 \leq \theta \leq \pi/4$.

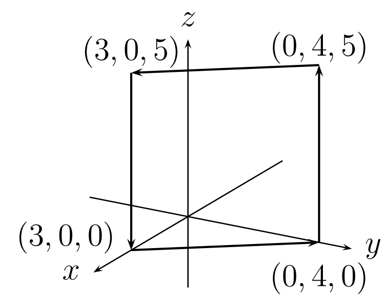


3 (10 points)

Evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r},$$

where $\mathbf{F}(x, y, z) = \langle e^x + z, \cos y + xy, z^2 + y \rangle$ and C is the curve which is the union of four lines; starting at $(3, 0, 0)$, going to $(0, 4, 0)$, then to $(0, 4, 5)$, then to $(3, 0, 5)$, and back to $(3, 0, 0)$. Do this without calculating four line integrals.



4 (10 points) Evaluate

$$\iiint_D \sqrt{x^2+y^2}\,dV,$$

where D is the region between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$; and where $z \geq 0$ and $x \geq 0$.

5 (10 points) Evaluate

$$\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S},$$

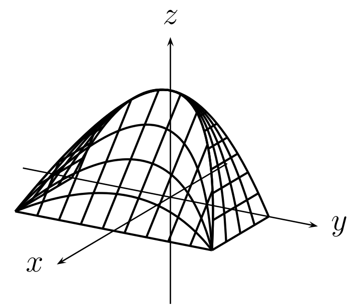
where $\mathbf{F}(x, y, z) = \langle zy, 2zx, xy \rangle$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 25$ below the plane $z = 4$. The surface is oriented inward towards the origin. Do this without integrating over the surface of the sphere.

6 (10 points)

Evaluate

$$\iint_S \mathbf{F} \cdot d\mathbf{S},$$

where $\mathbf{F}(x,y,z) = \langle x + \cos(y), e^z - y, zy^2 \rangle$ and S is the surface of the region D bounded by $z = 1 - y^2$, $x + z = 1$, $x = 0$, and $z = 0$. See the figure to the right.



7 (10 points) Evaluate

$$\iint_S \mathbf{F} \cdot d\mathbf{S},$$

where $\mathbf{F}(x, y, z) = \langle x, x^2 z, zy \rangle$, and S is the surface of the cylinder $x^2 + z^2 = 4$ between $y = -2$ and $y = 2$ oriented away from the y-axis (outward). This *does not* include the "lids" of the cylinder.

Some Useful Formulas

$\text{grad} f = \nabla f$, $\text{div}\mathbf{F} = \nabla \cdot \mathbf{F}$, $\text{curl}\mathbf{F} = \nabla \times \mathbf{F}$

- $$\iiint_D f(x,y,z)\,dV = \iiint_{D'} f(r\cos\theta, r\sin\theta, z) r\,dz\,dr\,d\theta$$

- $$\iiint_D f(x,y,z)\,dV = \iiint_{D'} f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$$

- $$\int_C f\,ds = \int_a^b f(\mathbf{r}(t))\|\mathbf{r}'(t)\|\,dt$$

- $$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)\,dt = \int_C \mathbf{F}\cdot\mathbf{T}\,ds$$

- $$\iint_S f(x,y,z)\,dS = \iint_D f(\mathbf{r}(u,v))\|\mathbf{r}_u\times\mathbf{r}_v\|\,dA$$

- $$\iint_S \mathbf{F}\cdot d\mathbf{S} = \iint_S \mathbf{F}\cdot\mathbf{n}\,dS = \iint_D \mathbf{F}\cdot(\mathbf{r}_u\times\mathbf{r}_v)\,dA$$

- Green's Theorem
$$\int_{\partial D} P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\,dA$$

- Stokes's Theorem
$$\int_C \mathbf{F}\cdot d\mathbf{r} = \iint_S (\nabla\times\mathbf{F})\cdot d\mathbf{S}$$

- The Divergence Theorem
$$\iint_S \mathbf{F}\cdot d\mathbf{S} = \iiint_D \nabla\cdot\mathbf{F}\,dV$$

- $\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$, $\quad \sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos(2\theta)$