Structural Engineering Modeling Issues

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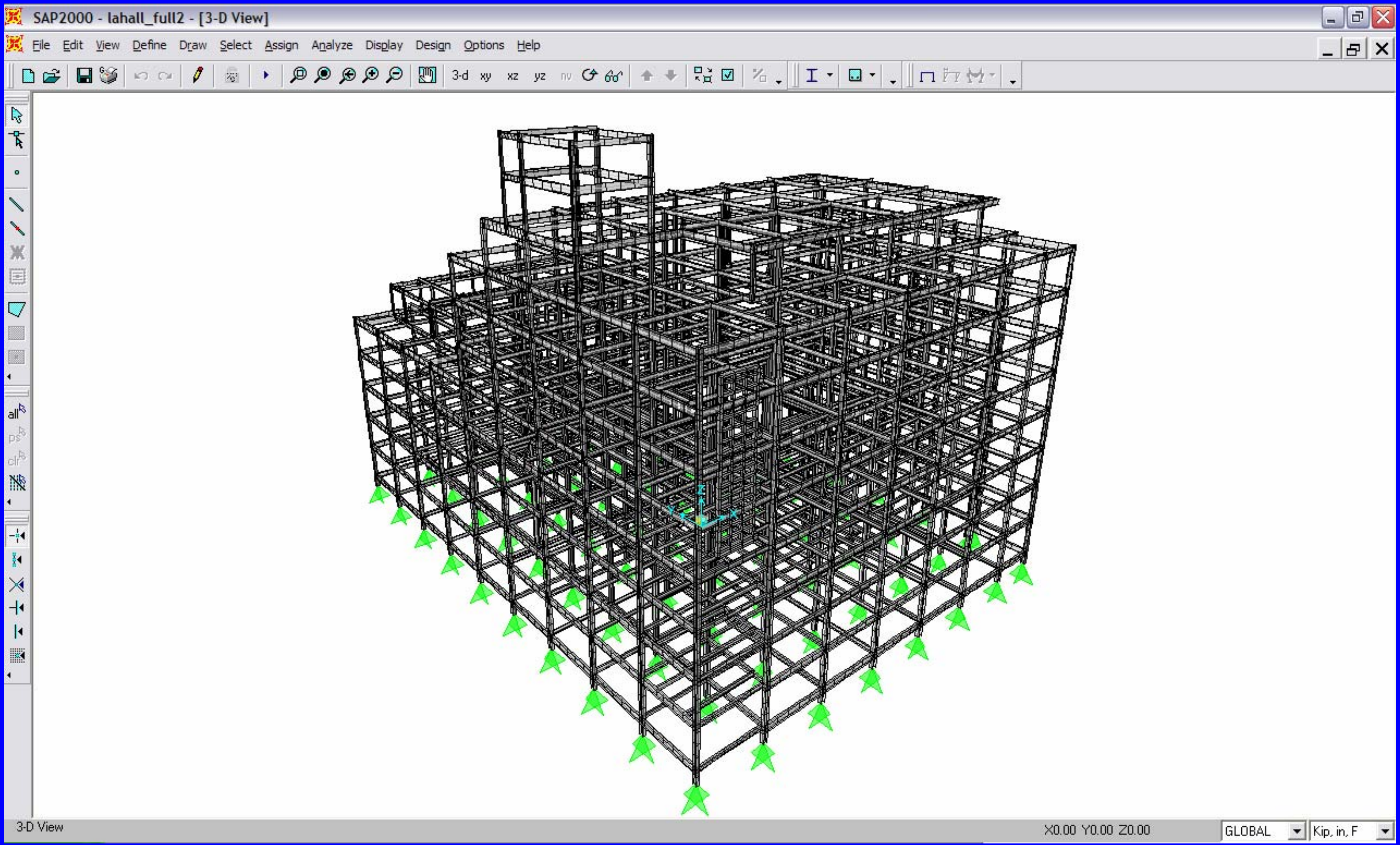
Extraction of Structural Matrices

$$M \cdot \ddot{u} + C \cdot \dot{u} + K \cdot u = f$$

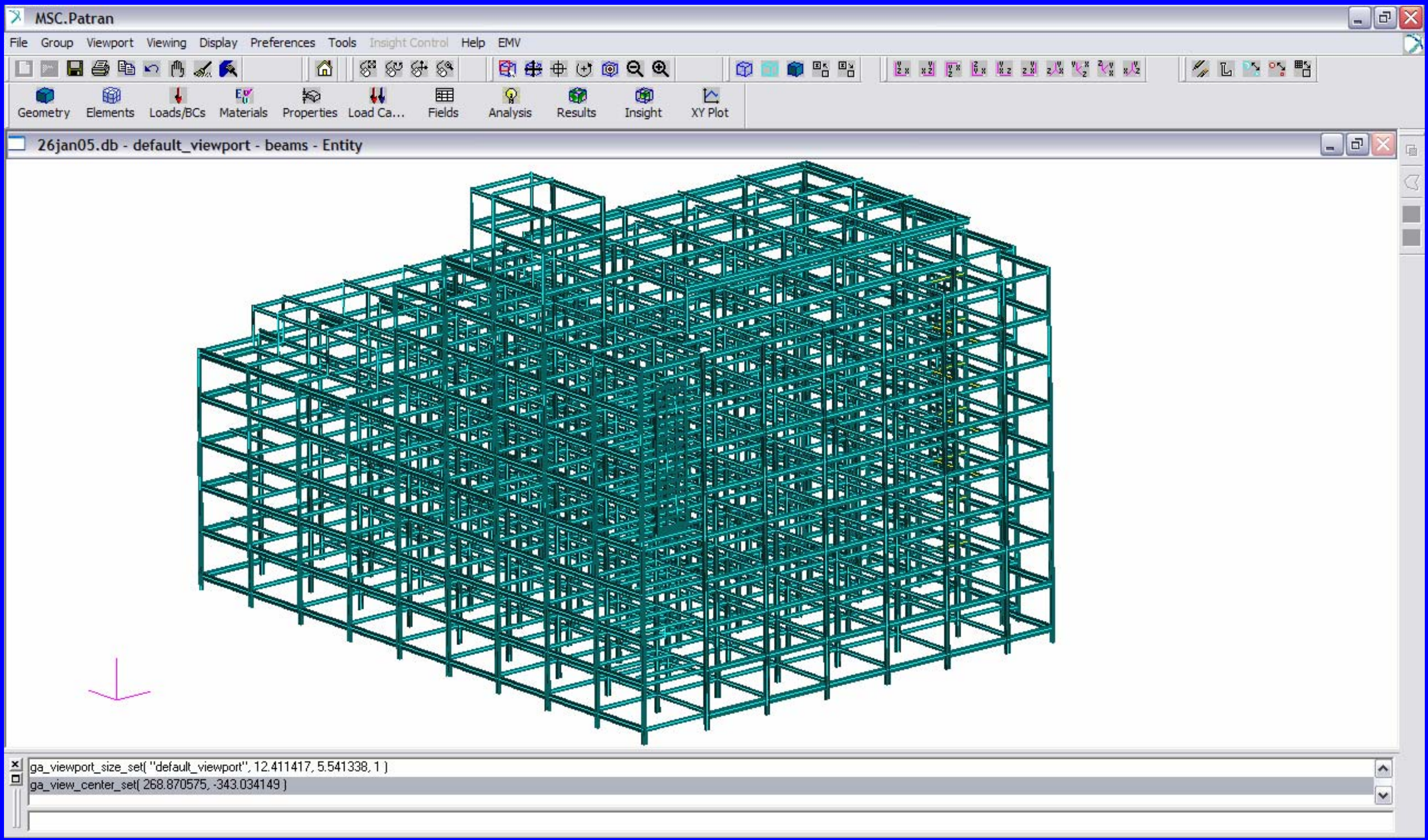
Difficulty arises due to the fact that most structural mechanics finite element analysis packages do not allow the user to output the Mass(M), Damping (C), and Stiffness (K) matrices.

Nastran was chosen as a tool due to its flexibility for outputting the structural matrices through its user interface module.

Sap2000 Model of the Building



Converted Nastran model of the Building



Nonlinear Response

$$M \cdot \ddot{u}(t) + C \cdot \dot{u}(t) + K(u,t) \cdot u(t) = f(t)$$

Due to the presence of nonlinear behavior, linear superposition principles do not apply.

Modal analysis can not be used for nonlinear response.

In addition, frequency response analysis is not possible. The solution has to be done in the time domain.

The only applicable method is direct transient response analysis in the time domain.

Modal Dynamic Analysis

$$M \cdot \ddot{u} + C \cdot \dot{u} + K \cdot u = f$$

$$u(t) = P \cdot X(t)$$

$$\tilde{M} \cdot \ddot{X}(t) + \tilde{C} \cdot \dot{X}(t) + \tilde{K} \cdot X(t) = \tilde{f}(t)$$

$$\tilde{M} = P^T \cdot M \cdot P \qquad \tilde{C} = P^T \cdot C \cdot P \qquad \tilde{K} = P^T \cdot K \cdot P \qquad \tilde{f} = P^T \cdot f$$

Mode shapes obtained from undamped free vibration:

$$M \cdot \ddot{u} + K \cdot u = 0$$

$$u = \phi \cdot \sin \omega (t - t_0)$$

$$K \cdot \phi = \omega^2 M \cdot \varphi$$

$$\Phi = [\phi_1 \quad \phi_2 \quad \dots \quad \phi_n]$$

$$\Omega = \begin{bmatrix} \omega_1^2 & & & \\ & \omega_2^2 & & \\ & & \ddots & \\ & & & \omega_n^2 \end{bmatrix}$$

$$K \cdot \Phi = \Omega^2 \cdot M \cdot \Phi$$

$$\Phi^T \cdot K \cdot \Phi = \Omega^2$$

$$\Phi^T \cdot M \cdot \Phi = I$$ (mass normalized)

$$\ddot{X}(t) + \Phi^T \ C \ \Phi \cdot \dot{X}(t) + \Omega^2 \cdot X(t) = \Phi^T \cdot f(t)$$

Damping Effects

Modal damping:

$$\phi_i^T \cdot C \cdot \phi_i = 2\ \omega_i\ \xi_i\ \delta_{ij}$$

$$\ddot{x}_i(t) + 2\ \omega_i\ \xi_i\ \dot{x}_i(t) + \omega^2\ x_i(t) = f_i(t)$$

Rayleigh damping (mass and stiffness proportional):

$$C = \alpha\ M + \beta\ K$$

* Nastran allows modeling viscous damping effects by using discrete damper elements.

Viscous and Structural Damping. Two types of damping are generally used for linear-elastic materials: viscous and structural. The viscous damping force is proportional to velocity, and the structural damping force is proportional to displacement. Which type to use depends on the physics of the energy dissipation mechanism(s) and is sometimes dictated by regulatory standards.

The viscous damping force f_v is proportional to velocity and is given by

$$f_v = b\dot{u} \qquad \text{Eq. 2-6}$$

where:

b = viscous damping coefficient

$\dot{u}$ = velocity

The structural damping force f_s is proportional to displacement and is given by

$$f_s = i \cdot G \cdot k \cdot u \qquad \text{Eq. 2-7}$$

where:

G = structural damping coefficient

k = stiffness

u = displacement

i = $\sqrt{-1}$ (phase change of 90 degrees)

For a sinusoidal displacement response of constant amplitude, the structural damping force is constant, and the viscous damping force is proportional to the forcing frequency.

NASTRAN USER MANUAL

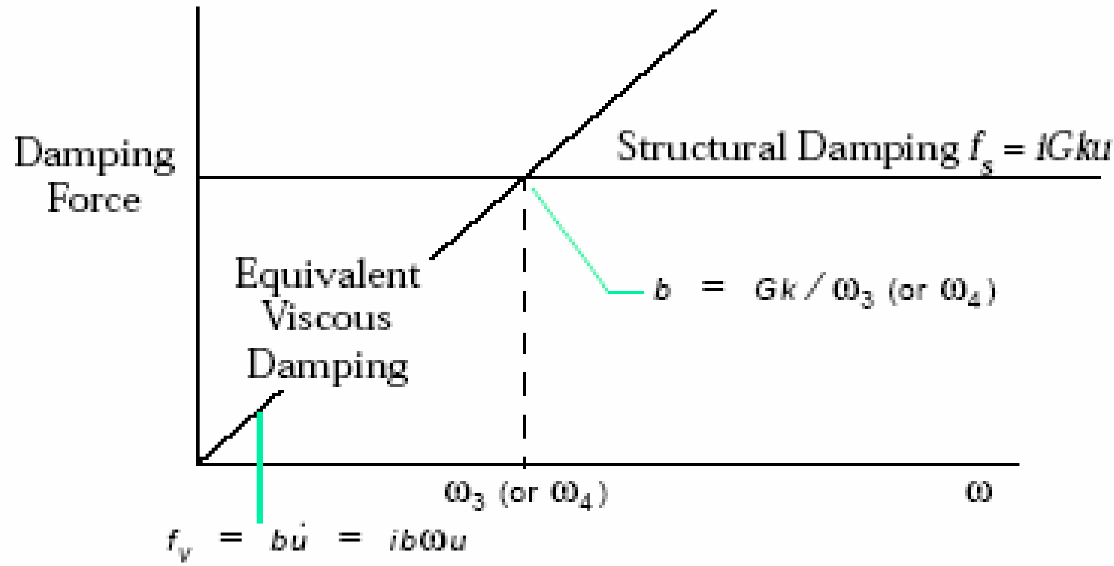


Figure 6-1 Structural Damping Versus Viscous Damping (Constant Oscillatory Displacement)

Transient Analysis - Implicit Time Integration

$$M \cdot \ddot{u}_{t+\Delta t} + C \cdot \dot{u}_{t+\Delta t} + K \cdot u_{t+\Delta t} = f_{t+\Delta t}$$

Linear acceleration assumption:

$$\dot{u}_{t+\Delta t} = \dot{u}_t + [(1-\delta) \cdot \ddot{u}_t + \delta \cdot \ddot{u}_{t+\Delta t}] \cdot \Delta t$$

$$u_{t+\Delta t} = u_t + \dot{u}_t \cdot \Delta t + [(1/2-\alpha) \cdot \ddot{u}_t + \alpha \cdot \ddot{u}_{t+\Delta t}] \cdot \Delta t^2$$

Compute effective stiffness matrix and force vector and solve for unknown displacements:

$$\widehat{K} \cdot u_{t+\Delta t} = \widehat{f}_{t+\Delta t}$$

$$LDL^T \cdot u_{t+\Delta t} = \widehat{f}_{t+\Delta t}$$

Implicit Time Integration (C’td.)

$$\widehat{K} \cdot u_{t+\Delta t} = \widehat{f}_{t+\Delta t}$$

$$\widehat{K} = K + a_0 \cdot M + a_1 \cdot C$$

$$\widehat{f}_{t+\Delta t} = f_{t+\Delta t} + M \cdot [a_0 u_t + a_2 \dot{u}_t + a_3 \ddot{u}_t] + C \cdot [a_1 u_t + a_4 \dot{u}_t + a_5 \ddot{u}_t]$$

Integration constants:

$$a_0 = \frac{1}{\alpha \cdot \Delta t^2} \qquad a_2 = \frac{1}{\alpha \cdot \Delta t} \qquad a_4 = \frac{\delta}{\alpha} - 1$$

$$a_1 = \frac{\delta}{\alpha \cdot \Delta t} \qquad a_3 = \frac{1}{2\alpha} - 1 \qquad a_5 = \frac{\Delta t}{2}\left(\frac{\delta}{\alpha} - 2\right)$$

Velocity and acceleration:

$$\dot{u}_{t+\Delta t} = \dot{u}_t + a_6 \cdot \ddot{u}_t + a_7 \cdot \ddot{u}_{t+\Delta t}$$

$$\ddot{u}_{t+\Delta t} = a_0 \cdot (u_{t+\Delta t} - u_t) - a_2 \cdot \dot{u}_t - a_3 \cdot \ddot{u}_t$$

$$a_6 = \Delta t \cdot (1 - \delta)$$

$$a_7 = \delta \cdot \Delta t$$