Part V

Math at Top Speed: Exploring and Breaking Myths in the Drag Racing Folklore

Rice University

Thursday, March 11, 2010 University of Texas



Resource Materials:

- NHRA Motorsports Museum
 - Steve Gibbs, Director
 - Greg Sharp, Curator
- Diamond P. Sports Inc.
- High Performance
 - Robert Post, Johns Hopkins University Press, 1994

Resource Materials (cont.):

- Top Fuel Handbook
 - Chris Martin, Mennonite Press, 1996
- NHRA 1999 Media Guide

Discussions or Support:

- Car Club Members
 - John Boyer
 - Bill Kelley
 - Jim Peterson
 - Chip Sweet

Family

- Richard Tapia, Jr
- Bobby Tapia
- Jean Tapia
- Becky Tapia

- Colleagues
 - Carl de Boor
 - Arturo Fuentes
 - Pol Spanos
 - Michael Trosset
 - Zhujin Wu
- Rice Students/Assistants
 Josef Sifuentes
 Hilena Vargas

Infrastructure support:

• Center for Excellence and Equity in Education, Rice University (CEEE)

Drag Racing

"The Greatest Show on Wheels" Motorsports: Country's Most Popular Sports

Drag Racing: Most Popular MotorSport Drag Racing and Rock & Roll

Born in America in the 50's and went crazy in the 60's

The Drag Race

- Kenny Berstein vs. Shirley "Cha-cha" Muldowney
- Lori Johns vs. Dick LaHaie

Drag Racing Terminology

Top Speed or MPH

The velocity (in mph) the dragster has reached at the end of the quarter mile (1320 feet).

Elapsed Time

The time taken (in seconds) by the dragster to travel the quarter mile.

• "Fast" describes a good top speed

"Quick" describes a good elapsed time

Dragster Facts

- One top fuel dragster 500 cubic inch Hemi engine makes more horsepower than the first four rows of stock cars at the Daytona 500.
- Under full throttle, a dragster engine consumes 1 ½ gallons of nitro methane per second; a fully loaded 747 consumes jet fuel at the same rate with 25% less energy being produced.
- A stock Dodge Hemi V8 engine cannot produce enough power to drive the dragster's supercharger.
- Dragsters reach over 100 mph in one second and 300 mph before you have completed reading this sentence.

The Machine Versus Nature (Gravity)

Elapsed Time Top Speed

Dragster World Record	4.43 sec	336 mph
Free Fall (1 g)	9.06 sec	198 mph

Unbelievable: In 1 second, a world class dragster is going 100 mph

The Birth of Drag Racing Early 1950's Beginning of an Era

Characterized by

- Experimentation
- Innovation
- Ingenuity
- Creativity

- Boldness
- Large Variances
- Informality
- Much Fun

Today: Sport very Professional and All Cars are the Same

Analogous Situation for Mathematics, Science, Etc.

The First Dragsters

The First "Rail Job"



Dick Kraft – Santa Ana, California, 1950 – 110 mph



Driver Behind Rear Axle

"First" Slingshot



Mickey Thompson –1954 – 138 mph



Engine Behind Driver

"The Pride of Tucson" World Top Speed and Elapsed Time Records



Lyle Fisher and Gary "Red" Greth Speed Sport Roadster –1957 – 169 mph Car dominated in 1960 – 8.03 ET (world record)

Sidewinders

Engine Sideways



Jack Chrisman –1959 – 152 mph (on GAS only)

Aircraft Engines

The Green Monster



Art and Walt Arfons –1959 – 170 mph

Jet Cars



Roger Harris –1970 – 275 mph

Twin-Engined

Two Side by Side Engines Are Part of Frame- Four Rear Wheels



Eddie Hill –1961 – 170 mph

Four Engines



"TV" Tommy Ivo –1960 – 165 mph

In spite of often attaining better times (both elapsed time and MPH), all designs were racing failures when compared to the single-engined slingshot. The cars were too bulky, too inflexible, not agile, too heavy. However, the slingshot became too dangerous.

The drivers, sitting behind the machinery, would be critically hurt or killed when machinery exploded (engines, superchargers, rear-ends). In 1970 after a serious accident Don Garlits designed a car that put the driver in front of the "noise and machinery". This turned out to be today's dragster. Not a slingshot, but close to it.

Slingshots Becomes Too Dangerous



Don Garlits –1970

"Back to the Future" Today's Dragsters



The Budweiser King

Many Dimensions of Danger

At the Edge of the Laws of Physics

A Piece of Americana

Our Humble Beginnings Circa 1957












David and Goliath Revisited

In 1959 in a highly publicized match race, Bobby in the little Chevy *Almost Grown* beats Art Arfons in the *Green Monster*.

"Bobby The Giant Killer"



Bobby Becomes World Class

Long Beach Strip Record July 10, 1965 209.78 mph





Bobby Sets World Elapsed Time Record



* * * JOURNAL OF THE NATIONAL HOT ROD ASSOCIATION

5.54 C HL-

Orange County Meet Boasts Quickest Yet

EBRUARY 16, 1968

<text><text><text><text><text><text><text><text><text><text><text><text><text><text><text><text><text><text>

Naver Pool (the entry nor gam and the latest M& HS, TM shows a

* * *

25c

VOL. VIII -- NO. 50

February 1968

October 4, 2002 Bakersfield, California

Bobby Tapia Inducted into the NHRA Hall of Fame

Part II

Some Mathematical Insights

Myth Number One

Since clocks give "average speed" in the time trap, dragsters are really going faster (today by about 2-3 mph) at the end of the quarter mile than indicated by the clock speed.

Mathematical Study Number One

What is the true speed (as opposed to clocked speed) that today's dragsters achieve in a quarter mile?

Historical Fact

Considerable attention is given to the driver who first reached a major top speed milestone e.g., 200 mph, 300 mph, 400 mph, etc.

Errors in the Calculation of Top Speed

Truncation error (mathematical error)
Clock placement error (human error)
Clock resolution error (mechanical error)

Mathematical Truncation Error

Let s(t) denote the dragster trajectory; i.e., we write distance as a function of time.

Distance	s(t)
Velocity (speed)	s'(t)
Acceleration	s''(t)
Actual top speed	s'(t(1320))

Top speed is calculated by placing clocks at 1254 feet and 1320 feet and using the difference quotient.

 $s'(t(1320)) \approx \frac{s(t(1320)) - s(t(1254))}{t(1320) - t(1254)}$

- To a mathematician, this is a first-order backward difference approximation to the derivative, i.e. the scope of the 2-point linear interpolant.
- To a physicist, this is simply the average speed* in the time trap.

* Average speed means the constant speed needed to perform the task.

Theorem

If acceleration is positive in the timetrap, then dragsters are indeed going faster than their clocked speed.

Proof: Taylor's Theorem

The Reality of the Situation

Higher order polynomial interpolation shows that while mathematical truncation error is, in general, positive, it is negligible and safely less than ¹/₂ mph.

Error in Clock Placement

Easily derived formula:

$\text{Error (mph)} = \frac{\text{Trap Error (feet)} \times \text{mph (clocked)}}{66}$

If mph (clocked) is 330 mph (the world record) and the trap error measurement is $\frac{1}{8}$ of an inch, then

Error (mph) = .05

Conclusion:

Clock placement error, like truncation error, is also quite negligible.

Error From Limited Clock Resolution

Clock resolution is .001 seconds; therefore, maximum clock error is .002 seconds (two clocks). **Easily Derived Sharp Error Bounds for mph (Actual) in Terms of mph (Clocked)**

 $\frac{45 \text{ mph (clocked)}}{45 + .002 \text{ (mph (clocked))}}$

 \leq mph(actual) \leq

 $\frac{45 \text{ mph (clocked)}}{45 - .002 \text{ (mph (clocked))}}$

Example

For 300 mph (clocked) we have:

$296.05 \le mph (actual) \le 304.04$

- Very significant error (up to 4 mph).
- For 1 mph accuracy at 300 mph, the clocks would have to have a resolution accuracy of .0005 seconds.

Was Kenny Bernstein the First to * Reach the Milestone of 300 mph?

Sept. 9, 1990	Gary Ormsby
Prob = 0.00	296.05 [292.21, 300.00]
March 12, 1992	Kenny Bernstein
Prob = .12	296.93 [293.06, 300.90]
March 12, 1992	Mike Dunn
Prob = .14	297.12 [293.25, 301.10]

Who First Reached the Milestone * of 300 mph? (cont.)

March 20, 1992	Kenny Bernstein
Prob = .72	301.70
	[297.71, 305.80]
October 30, 1993	Scott Kallita
Prob = 1.00	308.64
	[304.46, 312.93]

Answer: Maybe Ormsby, Bernstein, or Dunn, but Kallita certainly ran 300 mph or better. Mythological Concern Number Two (Early Drag Racing Confusion) How Can a Slower Car Beat a Faster Car?

The Harsh Reality

As far as winning the race goes, top speed is meaningless, elapsed time (quickness) tells it all.

"Quick vs. Fast" In the early days, elapsed time was not measured.

Mathematical Study Number Two

Quick vs. Fast

We will use calculus to generate mathematical understanding and theory that explains and validates the belief that the critical component in wining a drag race (i.e. promoting quickness) is initial acceleration. This study will lead us to "The Fundamental Theorem of Drag Racing".

Historical Comment

Throughout the history of drag racing, there has existed an infatuation with—and a mystique about—top speed, but not elapsed time. Indeed, in the early days of drag racing (1950) there were only top speed clocks and no elapsed time clocks. In part, this infatuation and mystique comes from the fact that top speed is easier to understand and appreciate. This appreciation was strongly nurtured and promoted by America's (pre-drag racing), involvement and success in the quest for land speed records on the legendary Great Salt Lake courses at Bonneville, Utah (and other dry lakes like El Mirage in California). 67

World Land Speed Record



Mickey Thompson –1961 – 406 mph



Quick vs. Fast

Illustrative Example:

Bobby and the Green Monster



Basics on Elapsed Time Versus Top Speed
Let s(t) denote the dragster trajectory; i.e., we write distance as a function of time.

Distance	S(t)
Velocity (speed)	s'(t)
Acceleration	s''(t)
Actual top speed	s'(t(1320))

Dragster – Jet Car Comparison



Observations

The total area under the acceleration curve is equal; hence velocity at the end of the race is nearly somewhat the same. However, the car with fast initial acceleration spends most of its time going fast, so the area under the dragster velocity curve (i.e. distance traveled for a fixed time) is much greater and therefore it gets to the finish line first. So, it is not just area under the acceleration curve that is critical, but that the area is large for small values of time (i.e. near the start).

Relevant Basic Questions

Assume that the dragster acceleration is positive in the time interval in question. Suppose that a positive (small duration) pulse input is added to the acceleration at a specified time.

Question 1: When should the pulse input be added in order to minimize the quarter-mile elapsed time?

Question 2: When should the pulse input be added in order to maximize the quarter-mile top speed?

Fundamental Theorem of Drag Racing

- i. The elapsed time is uniquely minimized by adding the pulse input as early as possible, i.e. at time zero.
- ii. The top speed is uniquely maximized by adding the pulse input as late as possible, i.e. nearest final time.Equivalently top speed is maximized when elapsed time is slowest.

Proof Idea

Proof follows from arguments that exploit the monotonicity of velocity and distance as functions of time.

Proof

Without loss of generality, let the pulse input be the indicator function of the interval $\begin{bmatrix} t_0, t_0 + \Delta t \end{bmatrix}$

We use hats to denote new quantities, i.e. quantities after the pulse has been added.



New velocity: $\hat{s}'(t) = s'(t) + \Delta t$ for $t \ge t_0 + \Delta t$ New distance: $\hat{s}(t) = s(t) + t - t_0 - \frac{1}{2}\Delta t$ Observation: $\hat{s}'(t)$ does not depend on t_0 $\hat{s}(t)$ increases as t_0 decreases

Proof: (Part i)

Final elapsed time:

$$\hat{s}(\hat{t}_F) = s(\hat{t}_F) + \hat{t}_F - t_0 - \frac{1}{2}\Delta t = 1320.$$

- Observe: $s(\hat{t}_F) + \hat{t}_F$ decreases/increases with t_0 By monotonicity: $s(\hat{t}_F)$ decreases/increases with \hat{t}_F
- Hence: \hat{t}_F decreases/increases with t_0 Conclusion: $t_0 = 0$ uniquely minimizes \hat{t}_F .

*

Proof: (Part ii)

Final velocity:

$$\hat{s}'(\hat{t}_F) = s'(\hat{t}_F) + \Delta t$$

- Since s''(t) > 0, we have s'(t) is monotone increasing.
- Hence $\hat{s}'(t)$ is also monotone increasing.
- Therefore $\hat{s}'(\hat{t}_F)$ will be uniquely maximized when \hat{t}_F is maximized as a function of t_0 , i.e. when $t_0 + \Delta t = \hat{t}_F$.

*

Conclusion

Our fundamental theorem takes a big step towards explaining why fast cars are often not quick and quick cars are often not fast. The two actually work against each other. In our model slowest elapsed time gives fastest top speed. In order to win maximize your initial acceleration.

Translation to Another Form of Racing

If you want to win a short foot race, then don't sprint (accelerate) at the end, accelerate as much as you can at the start and be totally undone at the end of the race, and just hang on. Example: Ben Johnson vs. Carl Lewis

Remark

Webster I:

Drag race – "a speed contest between automobiles".

He missed the essence of drag racing, "speed" should be replaced with "acceleration". This has been done in Webster II.

Myth Number Three

In the early days of drag racing there was an implied belief that one g-force (freefall) is the limit to how fast a car can accelerate. Modern Day Support of the Implied Belief



"It goes from zero to 60 in a little under 3 seconds."

Cartoon by Sidney Harris appeared in *Physics* by P.A. Tipler (1991) Historical Background and the Proof of the 1g Limit

Feb. 7, 1953:Lloyd and Art Chrisman clock a quarter-mile performance of 140.08 mph in 9.40 seconds.

May, 1953:

Navarro in Rods and Customs

"The maximum speed, as calculated by formula, that any internal combustion engine powered vehicle driving through its wheels will ever attain is 167 mph. Since 100% efficiency is virtually impossible, the fact that 140 was actually reached is almost unbelievable."

August, 1953:

Navarro in Rod and Custom

"A good number of readers have written asking just what this formula is, while others have challenged the statement. [Our consultant] Mr. O.L. Vosburg was kind enough to send us the formula and it is as follows." Vosburg's Derivation (of the one g limit)

Newton's Second Law of Motion

or

$$F = ma$$
$$a = \frac{F}{w}g$$

The coefficient of friction:

$$\mu = \frac{F_{\max}}{W} \le 1$$



"A reasonable upper bound for the coefficient of friction between rubber and asphalt is .70"

Hence

 $\mu \leq .70$

and standard formulas give



Navarro (in the same May 1953 article):

"We took Mr. Vosburg's letter to several engineers who immediately whopped out their slide rules, and, after a few minutes, came to the unanimous conclusion that the formula is, indeed, correct."

March, 1954:

Don Yates and Verne Mikkelsen turn in a quarter-mile performance of 144.85 mph in 8.99 seconds (better than the 1 g time of 9.06 seconds)

The drag racing community theoreticians (racers don't care) respond:

The clocks gave a bogus time; you can't beat 1 g.

The Yates and Mikkelsen time was not entered into the record books, and ironically we did not see another sub 9 second elapsed time for three years.

"Driving the Last Nail" The Pomona Experiment

In order to settle the 1-g controversy, in October of 1956, representatives from NHRA and the supplier of the timing clocks took over the Pomona, California drag strip for one day accompanied by three drag cars. They installed clocks at 132 foot intervals for the entire quarter mile (1320 feet) and timed the three cars with these clocks.

The Pomona Experiment (cont.)

The Nichols Brothers Quincy Auto Parts dragster covered the first 132 feet in 2.066 seconds.

Barney Navarro reports on the "Pomona Experiment"

... June 1957 issue of *Rod and Custom* Navarro's conclusion:

Using the formula $s = \frac{1}{2}at^2$ Navarro concludes that the Quincy Auto Parts dragster time of 2.066 seconds for the first 132 feet gives an "average" acceleration of 1.92 g's. The net effect is that the myth of a maximum acceleration of 1 g was effectively laid to rest.

Estimating Maximum Acceleration of Today's World-Class Dragsters

Our approach

Using the data given to us by NHRA for the world record run, interpolate the first n data points with s(t), a polynomial of degree *n*, requiring that s'(0) = 0. We estimate maximum acceleration by maximum of s''(t) in the time interval under consideration.

National Hot Rod Association (NHRA)

Elapsed time run data

60 ft	330 ft	660 ft	1000 ft	1254 ft	1320 ft
t(60)	t(330)	t (660)	t(1000)	t(1254)	t(1320)

Obvious additional information: Distance at t = 0 is zero. (s(0) = 0)Speed at t = 0 is zero. (s'(0) = 0)Collectively, we have 8 pieces of information. Plot of world record dragster run interpolating the NHRA data with a polynomial of degree 7.



Maximum Acceleration

Degree of polynomial	Maximum acceleration
2	5.07 g's
3	5.48 g's
4	5.76 g's
5	6.13 g's
6	6.55 g's
7	7.10 g's

Theorem

The estimate of 5.07 g-forces is a guaranteed lower bound for the true maximum acceleration.

Proof

For the case n=2 (quadratic polynomial) our procedure gives a curve of the form

$$s(t) = \frac{1}{2}at^2$$

The value of *a* is the constant acceleration needed to replicate the given 60 foot acceleration. Hence, it is a lower bound for the true maximum acceleration.

Comments

In all cases the maximum acceleration occurred at time t = 0. Remember our findings in the study of quick versus fast. It should not be surprising that we have seen that the perfect elapsed time machine must have extremely large acceleration for small values of time. A maximum acceleration of 7-8 g's seems quite reasonable.

Hard to Believe

If

and

 $a = \mu g = 7g$, $\mu = \frac{F_{\text{max}}}{1} = 7$ then it follows that ${\mathcal W}$ $F_{\rm max} = 7w.$

Can it be that the downward force exerted by the tires on the asphalt is seven times the weight of the dragster?

In the Direction of Validating Our Findings

The tires are compressed substantially at the start.



Something is Wrong with Vosburg's Theory

• Is it Newton's second law of motion?

• Is it the coefficient of friction theory?

Our Explanation of Acceleration Greater than One g-force

Let's go back to first principles and identify factors contributing to μ , the coefficient of friction

$$\mu = \frac{F_{\max}}{w}$$
Factors Contributing to Large Coefficient of Friction

Category I: Traction enhancement factors

- Non-smooth irregular asphalt surface
- Soft spongy wide tires
- Low tire pressure increasing the contact surface
- The burn-out (hot, gooey, and sticky tires)

These four factors create a tire/asphalt geardrive effect that increases the value of μ .

The Gear-Drive Effect

(arbitrarily large μ)



Factors Contributing to Large Coefficient of Friction (cont')

Category II: Weight-Force enhancement factors

- Aerodynamic factors (negative lift from wing)
- Rotation/translation factors (lifting of front end)

Remark: Downward forces effectively increase the weight of the dragster which allows, for a given traction environment, the utilization of more engine force, which in turn increases μ with respect to the static weight of the dragster.

Remark

Unfortunately, the net effect of the Vosburg 1 g controversy was to cause the drag racing community to (somewhat happily) have no faith in mathematics or physics, or people who practice such useless things.

Myth Number Four

Acceleration for a dragster is decreasing in time (starts large and gets smaller).



Mathematical Study Number Four

What is the Dragster Acceleration Profile?

Dragster Acceleration Profile

In the early days of drag racing it was believed (and was probably true) that the dragster's best rate of acceleration took place at a reasonable distance from the starting line.



This is certainly the case for your everyday car.

Barney Navarro, 1957

"The impossibility of [greater than] 1 g acceleration wasn't the only misconception exploded in the [Pomona] drag strip experiment. Examination of the elapsed times and average speeds brought out the startling fact that every car accelerated best in the first 132 feet. Without exception the acceleration got worse and worse the farther each car got from the starting line."

Barney Navarro (cont.)

Navarro's claim:

Contrary to popular belief, the acceleration for a dragster is decreasing in time (starts large and gets smaller).



Barney Navarro (cont.)

Navarro's "proof":

The differences between the average speeds corresponding to adjoining 132 foot intervals gets smaller and smaller as you move down the strip. Hence the acceleration must be decreasing.



*

Current Belief Held by Drag Racing Community

The acceleration for a dragster is largest at the start, and then decreases monotonically to essentially zero at the end of the quarter mile.



Our Investigative Approach to Acceleration Profile

Fit a smooth curve to the 8 pieces of NHRA data and then study the second derivative (acceleration) curve.

Polynomial of Degree 7



Acceleration not monotone

There is a blip.

C² Cubic Spline



Even the cubic spline shows a blip!

Superinterpolator: C⁴ Quintic Spline



The blip is still there!

The experts response:

 The blip should not be there. There is noise in the data, or the blip is an artifact of the interpolation scheme.

Our response:

No, after considerable experimentation we are convinced that the blip lives and is real, and we can explain it. This explanation will further validate the μ=7 phenomenon.

Recall Factors Contributing to Large Coefficient of Friction

Traction enhancing

- Non-smooth irregular asphalt surface
- Soft spongy wide tires
- Low tire pressure
- The burn-out (hot, sticky tires)

Weight enhancing

- Aerodynamic factors
- Rotation/translation factors

Back to Explaining the Blip

- Write the force exerted on the asphalt by the tires as
- Force: $F = F_o + F_E$
- where F_o , the ordinary force, represents the force that would result if none of the enhancement factors were present and F_E , the extraordinary force, represents the contribution to the force resulting from these enhancement factors. Then

Back to Explaining the Blip (cont.)

Acceleration:
$$a = a_o + a_E$$

where: $a_o = \frac{F_0}{W}g$ and $a_E = \frac{F_E}{W}g$
and

Coefficient of friction: $\mu = \mu_0 + \mu_E$ where $\mu_0 = \max \frac{F_0}{W}$ and $\mu_E = \max \frac{F_E}{W}$ **Observation:** $a_o \leq g$ (remember Vosburg)

The Acceleration Blip: Explained





128

The blip results because $a_E = \frac{F_E}{W}$ is so incredibly large at the start and decreases fast.

World Record Run

a_O and a_E acceleration components displayed



130

OK, The Blip is Real, Now What Does it Signify?

- **Recall:** $a(t) = \frac{F(t)}{W}g$
- **Consequence:** $\ddot{F}(t)$ has a blip at exactly the same time a(t) has a blip.
- **Question:** What causes a decrease in effective force even when the engine is running good?
- Answer: Excessive wheel spinning (drives F(t), a(t) and $\mu(t)$ to zero very quickly).
- **Conjecture:** The blip signifies the time point where the wheels stop spinning excessively and traction is restored.

Question: Do all drag cars have blips?

Answer: Yes, if they can spin their wheels excessively.

Bottom Line: You must spin to win.

Question: Was Navarro correct? Did the Quincy auto parts car not have a blip in its acceleration profile? Did cars not spin their wheels enough in those days to have a blip? Was Navarro's tool too crude to pick up the blip?

Let's see

Quincy Auto Parts 1956 Pomona Run



The blip is there (Navarro's approach smoothed the acceleration curve).

What About Vosburg?

We claim that $\mu = \mu_0 + \mu_E$. Vosburg made the mistake of ignoring μ_F (equivalently setting $\mu_F = 0$). His theory is fine (indeed, it is Newton's second law of motion). Moreover the assumption that $\mu_0 \leq 1$ is OK. However, for today's dragsters μ_{E} may be as large as 7; and in general is unbounded.

*

Where Do We Go From Here?



Quicker and Faster, of course!