

CAAM 552: Homework 1
Posted online on August 26
Due September 3 in class

Problem 1 (40 points)

Recall that the stiffness matrix is

$$\mathbf{A}_{i,j} = \int_0^1 \phi_i'(x)\phi_j'(x)dx$$

and the mass matrix is

$$\mathbf{M}_{i,j} = \int_0^1 \phi_i(x)\phi_j(x)dx$$

The ϕ_i 's are the hat functions defined on the partition $0 = x_0 < x_1 < \dots < x_N < x_{N+1} = 1$. Define $h_i = x_{i+1} - x_i$. Give an explicit expression of the stiffness and mass matrices (i.e. compute the entries for both matrices).

Problem 2 (30 points)

Explain what is wrong in both the variational setting and the classical setting for the problem

$$-u'' = f, \quad \text{with} \quad u'(0) = u'(1) = 0$$

That is, explain in both contexts why this problem is not well-posed.

Problem 3 (30 points)

Let $a(u, w) = \int_0^1 u'w'$. Let $\|u\|$ denote the L^2 norm of a function u ($\|u\| = (\int_0^1 u^2)^{1/2}$). Let $v \in \mathcal{C}^1(0, 1)$ such that $v(0) = 0$ and $a(v, v) < \infty$. Prove the following result

$$\|v\|^2 + \|v'\|^2 \leq Ca(v, v)$$

Give a value for the constant C . We say that the bilinear form a is coercive.