

CAAM 552: Homework 2
Posted online on September 8
Due September 15 in class

All problems are taken from the textbook.

Problem 1 (20 points)

If M is a closed subspace of a Hilbert space H , then prove that

$$(M^\perp)^\perp = M$$

Problem 2 (40 points)

Let $a(\cdot, \cdot)$ be the inner product for a Hilbert space V . Prove that the following two statements are equivalent for $\ell \in V'$ and an arbitrary closed subspace U of V :

- (a) $u \in U$ satisfies $a(u, v) = \ell(v) \quad \forall v \in U$
- (b) $u \in U$ minimizes $\frac{1}{2}a(v, v) - \ell(v)$ over $v \in U$

Problem 3 (40 points)

Let $a(\cdot, \cdot)$ be the inner product for a Hilbert space V . Let U be a closed subspace of V . For $g \in V$ define $U_g = \{v + g : v \in U\}$. (Note that $U_0 = U$). Prove that the following two statements are equivalent for arbitrary $g \in V$:

- (a) $u \in U_g$ satisfies $a(u, v) = 0 \quad \forall v \in U_0$
- (b) $u \in U_g$ minimizes $a(v, v)$ over $v \in U_g$