CAAM 552: Homework 4
Posted online on October 14
Due October 27 in class

All problems are taken from the textbook.

Problem 1 (20 points)

Construct nodal basis functions for $K =$ the rectangle with vertices $(-1,1),(1,-1),(1,1)$ and $(-1,-1)$, $\mathcal{P} = \mathbb{Q}_1$ and $\mathcal{N} =$ evaluation at vertices.

Problem 2 (20 points)

Prove that for $|\alpha| \leq m-1$ and for $u \in \mathcal{C}^{|\alpha|}(B)$:

$$D^\alpha T_y^m u(x) = T_y^{m-|\alpha|} D^\alpha u(x)$$

Problem 3 (30 points)

In class we showed that for all $u \in \mathcal{C}^m(\Omega) \cap W^{m,p}(\Omega)$:

$$\|R^m u\|_{L^\infty(\Omega)} \leq C(\mathrm{diam}(\Omega))^{m-n/p} |u|_{W^{m,p}(\Omega)}$$

Prove that the inequality holds true for all $u \in W^{m,p}(\Omega)$ by a density argument. Your proof cannot use Sobolev's imbedding theorem (also called Sobolev's inequality in textbook).

Problem 4 (30 points)

For a general bounded domain Ω with diameter $d = \mathrm{diam}(\Omega)$, define

$$\hat{\Omega} = \{(1/d)x : x \in \Omega\}$$

For $u \in W^{m,p}(\Omega)$ define $\hat{u}(\hat{x}) = u(d\hat{x})$. Show that

$$T_{d\hat{y}}^m u(d\hat{x}) = \hat{T}_{\hat{y}}^m \hat{u}(\hat{x})$$

and that

$$\hat{Q}^m \hat{u} = \widehat{Q^m u}$$