

Miscible Displacement in Porous Media

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Numerical simulations of the miscible displacement are presented. Three different schemes are considered, namely discontinuous Galerkin methods, mixed method coupled with method of characteristic and mixed method coupled with high order Godunov method. Both stable and unstable miscible displacement phenomena in heterogeneous porous media are investigated.

1. Introduction

Remediating groundwater pollution can be a challenging and costly environmental problem. The importance of numerical simulations is evident in developing reliable predictions of the transport of dissolved contaminants within the flow systems. In this paper, we study the miscible displacement phenomena, that is the displacement of one incompressible fluid (resident fluid) by another (solvent), in a porous medium Ω over the time interval I . The mathematical model arising from the conservation of mass and momentum, consists of a coupled system of equations:

$$-\nabla \frac{K}{\mu(c)} \nabla p \equiv \nabla \mathbf{u} = 0, \quad \text{in } \Omega \times I,$$

$$\phi \frac{\partial c}{\partial t} + \nabla \cdot (\mathbf{u}c - \mathbf{D}(\mathbf{u})\nabla c) = 0, \quad \text{in } \Omega \times I.$$

Here, p denotes the fluid pressure, c the solvent concentration, \mathbf{u} the Darcy velocity, K the permeability tensor, μ the fluid mixture viscosity, \mathbf{D} the molecular diffusion and mechanical dispersion tensor and ϕ the porosity of the medium. We assume the standard nonlinear relations for μ and \mathbf{D} [1]: $\mu(c) = \mu_0((1 - c_{\text{in}}c) + M^{-0.25}c_{\text{in}}c)^{-4}$ where μ_0 is a reference viscosity, c_{in} the injected concentration and M is the mobility ratio, that is the ratio of the resident fluid viscosity to the solvent viscosity; $\mathbf{D}(\mathbf{u}) = (\alpha_t|\mathbf{u}| + D_m)\mathbf{I} + (\alpha_l - \alpha_t)\mathbf{u}\mathbf{u}^T/|\mathbf{u}|$, where D_m, α_l, α_t are the molecular diffusivity, longitudinal and transverse dispersivities respectively. The following boundary conditions are associated with the system of partial differential equations:

$$p = p_0, \quad \text{in } \Gamma_D \times I,$$

$$\frac{K}{\mu(c)} \nabla p \cdot \mathbf{n} = 0, \quad \text{in } \Gamma_N \times I,$$

$$(\mathbf{u}c - \mathbf{D}(\mathbf{u})\nabla c) \cdot \mathbf{n} = c_{\text{in}}\mathbf{u} \cdot \mathbf{n}, \quad \text{in } \Gamma_+ \times I,$$

$$\mathbf{D}(\mathbf{u})\nabla c \cdot \mathbf{n} = 0, \quad \text{in } \Gamma_- \times I,$$

where Γ_+ (resp. Γ_-) denotes the inflow (resp. outflow) part of the boundary:

$$\Gamma_+ = \{\mathbf{x} \in \partial\Omega : \mathbf{u} \cdot \mathbf{n} > 0\}, \quad \Gamma_- = \partial\Omega \setminus \Gamma_+.$$

A collection of methods including standard Galerkin methods [2,3], mixed finite elements [4], modified method of characteristics [5–7] and high order Godunov method [8] have been used for solving the miscible displacement problem. The continuous Galerkin methods are not locally mass conservative. The mixed finite elements perform very well in the case of diagonal permeability and tensor product grids. However, the mixed methods are limited in treating full permeability tensors and highly unstructured grids. The usual characteristic and Godunov methods are used for approximating the concentration equation. They are of low order.

Recently, discontinuous Galerkin methods have gained a lot of attention from the finite element community. These methods have several appealing features: they conserve mass locally on each element, they support local approximations of high order, they are robust and local oscillations can be eliminated by the introduction of slope limiters, and they are implementable on unstructured and even non-matching meshes. Optimal convergence of these methods have been derived for single phase flow [9,10] and linear transport [11]. The goal of this work is to apply DG to the miscible displacement problem and to make numerical comparisons with two standard schemes, namely mixed finite element method coupled with characteristics method (CMM) and mixed finite element method coupled with high order Godunov method (HOG). The DG scheme used in this work is described in [12,13]. The first numerical comparisons between DG and mixed finite-high order Godunov [12,13], showed that the DG concentration fronts can be sharper than the HOG fronts. In this present work, comparisons with methods of characteristics known to be less diffusive than HOG, are given.

The rest of the paper is as follows. In section 2, the physical parameters are defined. The numerical results are presented in section 3. The last section contains some concluding remarks.

2. Numerical Settings

In all the numerical results, the computational domain $(0, 1600)^2$ is fixed. Dirichlet boundary conditions are imposed along the vertical edges ($p_0 = 1.e^8\text{Pa}$ for $x = 0$ and $p_0 = 0\text{Pa}$ for $x = 1600$). A unit concentration is injected at the inflow boundary (vertical edge for $x = 0$). The coarse mesh (h_0) consists of 8×8 squares. The refined meshes h_1, h_2 are obtained by successively refining the coarse mesh in an isotropic way. The molecular diffusivity is $D_m = 1.16e^{-9}m^2s^{-1}$, the dispersivities are $\alpha_l = 0.1m$ and $\alpha_t = 0.01m$. This yields a longitudinal Peclet number of 16000 and a transverse Peclet number of 160000, which means that the convective effects dominate the flow. The reference viscosity is $\mu_0 = 1cp$. Since it has been observed experimentally that the pressure varies much slowly than the concentration, we choose the time step for the concentration equation to be four times smaller than the time step for the pressure equation.

We vary two important parameters: the permeability field and the mobility ratio M . The mobility ratio is one of the factors that determine if the flow is stable or unstable. In the case of mobility ratio equal to unity, the fluid mixture viscosity is independent of

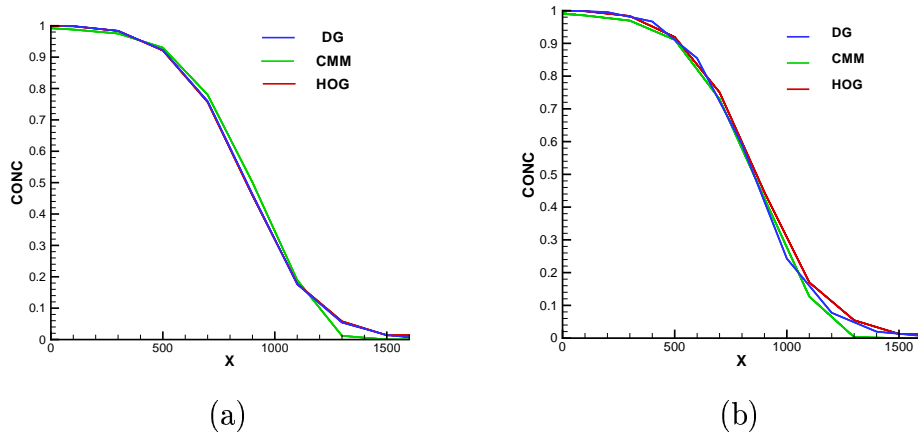


Figure 1. Concentration fronts: (a) $M = 1$ and $T = 1.4e^6$; (b) $M = 50$ and $T = 8.e^5$.

the solvent concentration, and the pressure and velocities field do not vary throughout the simulation. In the case of mobility ratio larger than unity, small instabilities in the flow will grow and the displacement is destabilized. In that case, formation of protusions of the solvent develop through the resident fluid. These unstable phenomena are referred to as viscous fingering [14].

The figures in the following section have been obtained by using the visualization package tool TECPLOT [15]. Since the HOG and CMM solutions are piecewise constants, linear interpolants are constructed in a postprocessing step. We use 11 contour levels ranging from 0 to 1 to represent the solvent concentration solutions.

3. Simulations

3.1. Uniform Permeabilities

We first consider the simple case where the permeability K is uniform and equal to $1.e^{-11}m^2$. Comparisons of concentration fronts are shown on Figure 1. We choose to approximate the DG concentration by piecewise discontinuous linear polynomials. All methods yield similar fronts. In the case where the mobility ratio is 1, the pressure is linear and constant over I , the velocity field is uniform $\mathbf{u} = (6.25e^{-4}, 0)ms^{-1}$. Figure 1(a) shows the fronts at the final time $t_0 = 1.4e^6s$. When the mobility ratio increases ($M = 50$), the solvent front is more advanced and the maximum concentration is reached in all the domain at t_0 . We show the concentration fronts at the time $t_1 = 8.e^5s$. This time, HOG and CMM differs slightly. The pressure field varies in time due to the coupling with the concentration equation.

3.2. Vertical Faults

A porous medium usually contains heterogeneities such as faults, shale lenses, pinch-outs. Those can be characterized by rapid changes in the permeability fields. In this example, we assume that there are vertical patches of low permeability (1000 times smaller

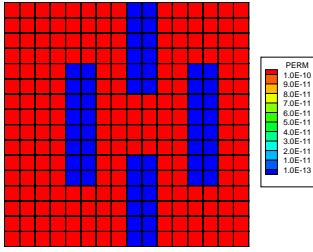


Figure 2. Permeability field on the refine mesh h_1 .

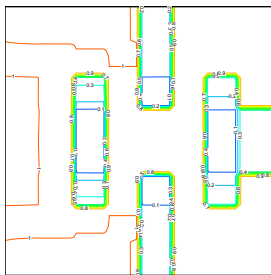
than the rock matrix permeability). The distribution of permeability is shown on Figure 2. The miscible displacement is simulated over $[0, 1.4e^6]$.

3.2.1. Stable Flow

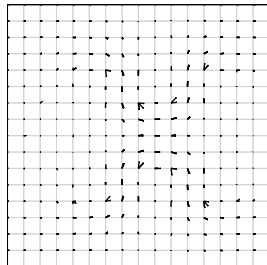
In the case of stable flow ($M = 1$), the concentration contours obtained on the coarse mesh are shown on Fig. 3.2.1. We can observe that the DG solvent concentration avoids very well the zones of low permeability. The HOG solution is more diffusive. The CMM solution is quite different. After refining the mesh twice (h_2), the HOG and CMM contours are similar to the DG contours on h_0 (Figure 4). The DG method yields sharper solutions because of the fact that the DG velocity field captures very well the heterogeneities of the medium. Figure 3.2.1 and Figure 4 also show the velocity fields for all three schemes on h_0 and h_2 type meshes. The grids shown on this figure are visualization, not computational grids.

3.2.2. Unstable Flow

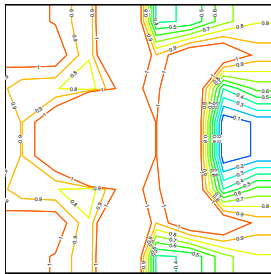
We now increase the mobility ratio ($M = 50$) and we observe that instabilities (or viscous fingering) occur. In that case, we stop the simulations at $t = 2.8e^5 s$. The first column of Figure 3.2.2 shows the concentration contours obtained with the three methods on coarse and refined meshes. On the coarsest mesh, the three approximations yield different results. The DG contours show that the solvent has avoided the regions of low permeability and has reached the outflow boundary. The CMM front has only advanced through half of the domain. The HOG front is faster than CMM's but still slower than DG's. In order to conclude, one has to compare the solutions obtained in the asymptotic range, this can be achieved by successively refining the grid. The more refined solutions are shown in the second column of Figure 3.2.2. It appears that the HOG solution is very close to the DG solution. However, there is more numerical diffusion associated with the HOG method than with the DG method. The CMM solution on the refined mesh seems to converge to the DG solution. The front has swept more than half of the domain. The differences between CMM and the two other methods can be explained by the fact that the asymptotic range has not been reached.



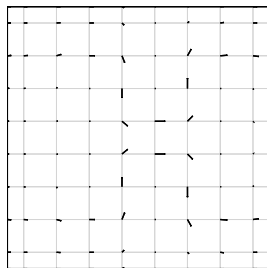
DG (a)



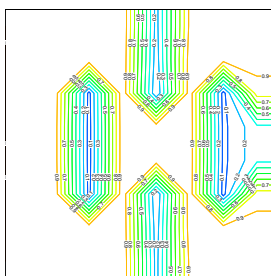
DG (b)



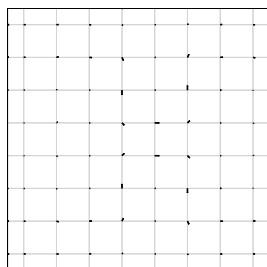
CMM (a)



CMM (b)

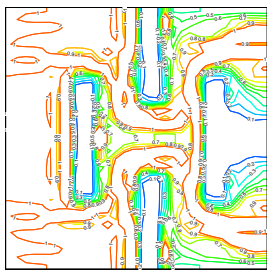


HOG (a)

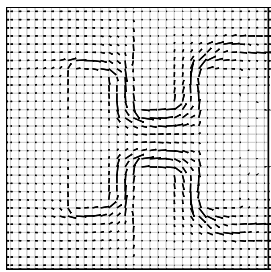


HOG (b)

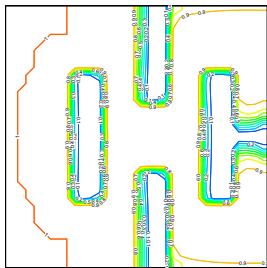
Figure 3. Concentration contours (a) and velocity fields (b) in the case of stable flow $M = 1$ on coarse mesh h_0 .



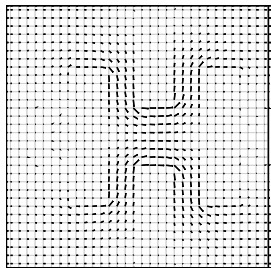
CMM (a)



CMM (b)



HOG (a)



HOG (b)

Figure 4. Concentration contours (a) and velocity fields (b) in the case of stable flow $M = 1$ on refined mesh h_2 .

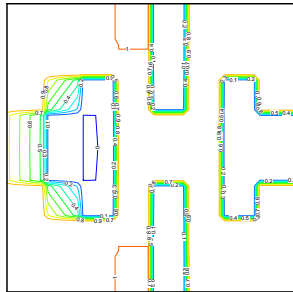
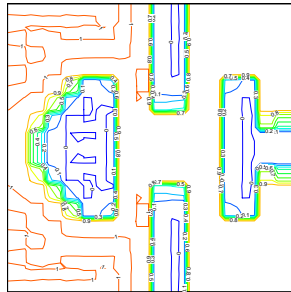
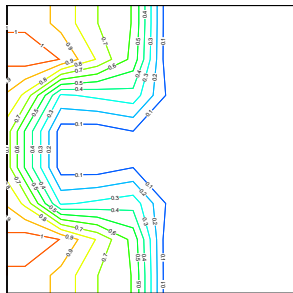
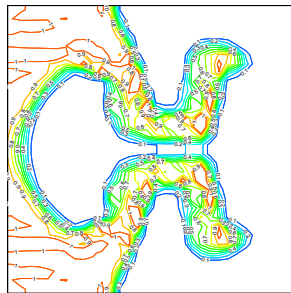
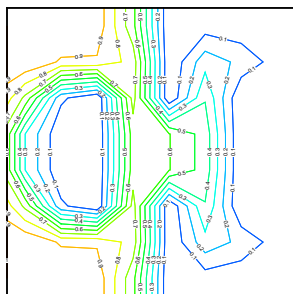
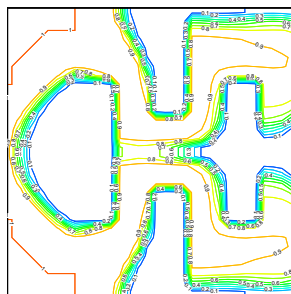
DG h_0 DG h_1 CMM h_0 CMM h_2 HOG h_0 HOG h_2

Figure 5. Concentration contours in the case of unstable flow $M = 50$ on coarse and refined meshes.

4. Conclusions

In this paper, we present numerical comparisons of miscible displacement for stable and unstable flow. These results show that DG is a competitive method to the mixed finite element method coupled with either characteristic method or high order Godunov method. Sharper and more accurate fronts are obtained with the discontinuous Galerkin method. The characteristic-mixed method yields poor solutions on the coarse mesh and in the case of unstable flow, one needs to refine the mesh several times, in order to obtain comparable solution to the high order Godunov or discontinuous Galerkin solutions.

REFERENCES

1. E.J. Koval, A method for predicting the performance of unstable miscible displacement in heterogeneous media, Soc. Pet. Eng. J. No. 3 (1963) 145.
2. R.E. Ewing and M.F. Wheeler, Galerkin methods for miscible displacement problems in porous media, SIAM J. Numer. Anal. No. 17 (1980) 351.
3. M.F. Wheeler and B.L. Darlow, Interior penalty Galerkin procedures for miscible displacement problems in porous media, Comp. Meth. Nonlinear Mech. (1980) 485.
4. J. Douglas, R.E. Ewing and M.F. Wheeler, A time-discretization procedure for a mixed finite element approximation of miscible displacement in porous media, R.A.I.R.O. Num. Anal. No. 17 (1983) 249.
5. R.E. Ewing, T.F. Russell and M.F. Wheeler, Convergence analysis of an approximation of miscible displacement in porous media by mixed finite elements and a modified method of characteristics, Comp. Meth. Appl. Mech. Engrg. No. 47 (1984) 73.
6. C.N. Dawson, T.F. Russell and M.F. Wheeler, Some improved error estimates for the modified method of characteristics, SIAM J. Numer. Anal. No. 26 (1989) 1487.
7. T.F. Russell, Time-stepping along characteristics with incomplete iteration for a Galerkin approximation of miscible displacement in porous media, SIAM J. Numer. Anal. No. 22 (1985) 970.
8. C.N. Dawson, Godunov-mixed methods for advection-diffusion equations in multidimensions, SIAM J. Numer. Anal. No. 30 (1993) 1315.
9. B. Rivière, M.F. Wheeler and V. Girault, Improved energy estimates for elliptic problems. Part I., Comp. Geosc. No. 3 (1999) 337.
10. B. Rivière, M.F. Wheeler and V. Girault, A priori error estimates for finite element methods based on discontinuous approximation spaces for elliptic problems, SIAM J. Numer. Anal. No. 39 (2001) 902.
11. B. Rivière and M.F. Wheeler, Nonconforming methods for transport with nonlinear reaction, Proceedings of the Joint Summer Research Conference on Fluid Flow and Transport in Porous Media, to appear.
12. B. Rivière, Discontinuous Galerkin methods for solving the miscible displacement problem in porous media, Ph.D Thesis, The University of Texas at Austin (2000).
13. B. Rivière and M.F. Wheeler, Discontinuous Galerkin methods for flow and transport problems in porous media, Com. Num. Meth. Engrg., to appear.
14. G.M. Homsy, Viscous fingering in porous media, Ann. Rev. Fluid. Mech. No. 19 (1987) 271.
15. AMTEC Data Visualization Software (<http://www.amtec.com>)