Drag Lecture 5a: Drag Force

▶ Computation of the Drag Force

Last time we computed the velocity and pressure of flow in the small Reynolds number limit. Today, we will compute the force of drag,

$$\mathbf{F} = \int_{\partial\mathcal{B}} \mathbf{\Pi n}\,\mathrm{d}S.$$

Recall that

$$\mathbf{\Pi} = p\mathbf{I} - \boldsymbol{\sigma}', \qquad \boldsymbol{\sigma}' = \eta(\operatorname{grad}\mathbf{v} + (\operatorname{grad}\mathbf{v})^T).$$

We will compute $\mathbf{\Pi n}$ on the surface of the sphere using the Symbolic Toolbox in MATLAB; see the `symdrag.m` code at the end of these lecture notes. The intermediate steps are messy, but the final formula is extremely clean. We assume that the system is oriented so that

$$\mathbf{u} = \begin{bmatrix} |\mathbf{u}| \\ 0 \\ 0 \end{bmatrix},$$

and thus find

$$\mathbf{\Pi n} = \begin{bmatrix} \frac{3}{2}|\mathbf{u}|\eta/\ell \\ 0 \\ 0 \end{bmatrix}.$$

The integrand in the drag force integral is thus constant, hence the integral is just a multiple of the surface area:

$$\mathbf{F} = \int_{\partial\mathcal{B}} \mathbf{\Pi n}\,\mathrm{d}S = \frac{3}{2}\frac{|\mathbf{u}|\eta}{\ell}(4\pi\ell^2)\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 6\pi|\mathbf{u}|\eta\ell\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

```
%
%  symdrag.m     Steve Cox, Feb 10, 2009
%
%  Compute the drag force on a sphere of radius "a" at low Reynold's
%  number using the Stoke's approximation developed in our
%  drag notes. In particular, we compute the integrand
%
%   Pin = p*n - sig*n   where n = -x/|x| is the normal into the sphere
```

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%                        p is the pressure and sig is the viscous
%                        part of the stress, sig = eta*(grad(v)+grad(v)')
%
%  we find that Pin = (3/2)(U*eta/a)*[1 0 0] is constant on the sphere
%  and so its integral over the sphere is simply (4/3)*pi*a^2*Pin
%

syms x1 x2 x3 U a eta

x = [x1; x2; x3];

X = x*transpose(x);

I=eye(3);

u = [U; 0; 0];

r = sqrt(x1^2+x2^2+x3^2);

v = u - (3/4)*(a/r)*(I+X/r^2)*u - (1/4)*(a^3/r^3)*(I-3*X/r^2)*u;

vx1 = diff(v,x1);
vx2 = diff(v,x2);
vx3 = diff(v,x3);

gradv = [vx1 vx2 vx3];

sig = eta*(gradv+transpose(gradv));

sigtn = simple(sig*(-x/r));

sigtns = simple(subs(sigtn,a,r))

p = -(3/2)*eta*(1/r^2)*transpose(u)*x;
pn = p*(-x/r)

Pin = simple(pn - sigtns)
```

[Steve Cox, 10 February 2009]